INFLUENCE OF KALMAN FILTER PARAMETERS TO THE INDUCTION MOTOR SPEED ESTIMATION

Krzysztof P. DYRCZ, Teresa ORLOWSKA-KOWALSKA
Wroclaw University of Technology
Institute of Electrical Machines, Drives and Measurements
ul. Smoluchowskiego 19, 50-370 Wroclaw, Poland
Phone: (+48 71) 320-35-46, Fax: (+48 71) 320-34-67,
E-mail: kdyrcz@imne.pwr.wroc.pl, tok@imne.pwr.wroc.pl

SUMMARY

The paper deals with the application of Kalman filter for state variable estimation of the induction motor. The mathematical model of the extended Kalman filter used for rotor flux and speed estimation was presented. Some simulation results were shown in the paper. The influence of parameters of the induction motor equivalent circuit on the state variable estimation quality was demonstrated. The problem of suitable choice of Kalman filter parameters, such as elements of the covariance matrix Q, was also discussed. The possibility of practical realisation of such Kalman filter was discussed.

Keywords: *Induction motor, speed estimation, flux estimation, Extended Kalman Filter*

1. INTRODUCTION

In regard to reliability of work, quality of realisation and comparatively low price, the induction motor (IM) has found a wide use in the drive systems. Recently is observed a great interest in drive systems without mechanical sensors, so-called *sensorless drives*. On the other hand, the development of microprocessor techniques and comparatively cheap signal processors (DSP) have enabled the realisation of advanced control methods of the induction motor. All these methods require the know-ledge of instantaneous values of the motor state variables, including the motor speed.

There are many methods of speed estimation based on the mathematical model of the induction motor [4],[7]. However, these methods are sensitive to changes of motor parameters, what causes significant estimation errors of state variables and thus incorrect work of the drive system [8].

From this reason, the usage of many simple algorithms of the state variables reconstruction is difficult, especially in the case of changes or not proper identification of the IM parameters. In such cases many authors propose the application of Kalman filters, which are specially devoted to the dynamical systems which operate in the presence of stochastic disturbances [1]-[3].

The algorithm of Kalman filter, like the other state estimators and observers, is based directly on the mathematical model of the induction motor, which significantly depends on the equivalent circuit parameters, changing during the drive operation or being badly identified.

So, the main goal of this paper is the answer to the question if the Kalman filter used for the rotor flux vector and angular speed estimation is more robust to the IM parameter changes than other algorithms used for this purpose [8]. In the paper the mathematical model of the extended Kalman filter used for rotor flux and speed estimation is presented and tested in simulations. The influence of parameters of the induction motor equivalent circuit on the state variable estimation quality is shown. The problem of suitable choice of EKF parameters, such as elements of the covariance matrix **Q**, is also demonstrated. The possibility of practical realisation of such Kalman filter is discussed.

2. INDUCTION MOTOR MATHEMATICAL MODEL

Differential equations for the electromagnetic variables of the induction motor, with usual assumptions [8], can be written in the following form of linear state equation [p.u]:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{x}(t) \tag{1}$$

where:

$$\mathbf{x}(t) = \left[i_{\alpha s}(t), i_{\beta s}(t), \Psi_{\alpha r}(t), \Psi_{\beta r}(t)\right]^T \text{ - state vector,} \tag{2}$$

$$\mathbf{y}(t) = \left[i_{\alpha s}(t), i_{\beta s}(t)\right]^T \text{ - out put vector,} \tag{3}$$

$$\mathbf{u}(t) = \left[u_{\alpha s}(t), u_{\beta s}(t)\right]^T \text{ - input vector,} \tag{4}$$

A, B, C – are respectively the state, control and output matrices.

In this model the additional assumption was made, e.g. the electromagnetic state variables - the stator current and the rotor flux have much faster dynamics than the mechanical state variable – the rotor speed. So the state equation is linear in respect to the state vector, but its state matrix consists of some elements which are speed-dependent and thus variable.

In such case, to perform the estimation of electro-magnetic as well as mechanical state variables using

Kalman filter, it is possible to extend the state vector by the additional element – the motor speed. In this case, the new, extended mathematical model of the induction motor will have a form of nonlinear state equation. According to the filtering theory [7], the noise signals should be added to the state and output vectors of the motor. So, with the above assumption, the extended model of the induction motor has the following form:

$$\dot{\mathbf{x}}_r(t) = \mathbf{f}[\mathbf{x}_r(t), \mathbf{u}(t), t] + \mathbf{G}(t)\mathbf{w}(t)$$
$$\mathbf{y}(t) = \mathbf{h}[\mathbf{x}_r(t), t] + \mathbf{v}(t) \tag{5}$$

where:
- new, extended state vector is:

$$\mathbf{x_r}(t) = [i_{\alpha s}(t), i_{\beta s}(t), \Psi_{\alpha r}(t), \Psi_{\beta r}(t), \omega(t)]^T = [x_{r1}, x_{r2}, x_{r3}, x_{r4}, x_{r5}]^T \tag{6}$$

- the extended state function of the motor:

$$\mathbf{f}[x_r(t), u(t), t] = \mathbf{A_r}(x_{r5})\mathbf{x_r}(t) + \mathbf{B_r}\,\mathbf{u}(t) \tag{7}$$

- the state matrix with elements dependent on chosen state variable:

$$\mathbf{A_r} = \begin{bmatrix} -\left(\frac{1}{\tau_s\sigma} + \frac{1-\sigma}{\tau_r\sigma}\right) & 0 & \frac{k_r}{\tau_r x_s \sigma} & \omega\frac{k_r\omega_b}{x_s\sigma} & 0 \\ 0 & -\left(\frac{1}{\tau_s\sigma} + \frac{1-\sigma}{\tau_r\sigma}\right) & -\omega\frac{k_r\omega_b}{x_s\sigma} & \frac{k_r}{\tau_r x_s\sigma} & 0 \\ \frac{x_M}{\tau_r} & 0 & -\frac{1}{\tau_r} & -\omega\omega_b & 0 \\ 0 & \frac{x_M}{\tau_r} & \omega\omega_b & -\frac{1}{\tau_r} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

- input matrix $\mathbf{B}_r$ and output matrix $\mathbf{C}_r$ are:

$$\mathbf{B_r} = \begin{bmatrix} \frac{\omega_b}{\sigma x_s} & 0 \\ 0 & \frac{\omega_b}{\sigma x_s} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{9}$$

$$\mathbf{C_r} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

where: $\mathbf{f}[\mathbf{x}_r(t), \mathbf{u}(t), t]$ – nonlinear function,
$\mathbf{u}(t)$ – as (4),
$\mathbf{G}(t)$ – the noise gain matrix,
$\mathbf{w}(t)$ – the state variable noise vector,
$\mathbf{v}(t)$ – the output signal noise vector,

and: $\tau_s = \frac{x_s}{R_s\omega_b}$, $\tau_r = \frac{x_r}{R_r\omega_b}$, $k_r = \frac{x_M}{x_r}$, $k_s = \frac{x_M}{x_s}$,
$\sigma = 1 - k_s k_r$, $\omega_b = 2\pi f_s$.

Next, this model should be written in the discrete form and the algorithm of the extended Kalman filter can be applied directly.

3. ALGORITHM OF EXTENDED KALMAN FILTER

The algorithm of the extended Kalman filter for the induction motor model (2) – (10) can be calculated in a few steps:

1) Computing of the state vector prediction:

$$\hat{\mathbf{x}}(k+1/k) = \hat{\mathbf{x}}(k/k) + T_s\mathbf{f}[\hat{x}(t_k/k), \mathbf{u}(k), k] \tag{11}$$

2) Estimation of filter covariance matrix:

$$\mathbf{P}(k+1/k) = \mathbf{\Phi}(k+1/k)\mathbf{P}(k/k) \times \times \mathbf{\Phi}^T(k+1/k) + \mathbf{Q}(k) \tag{12}$$

where:

$$\mathbf{\Phi}(k+1/k) = \exp\{\mathbf{F}(k)\mathbf{T_s}\} \cong \mathbf{I} + T_s\mathbf{F_k} \tag{13}$$

$$\mathbf{F_k} = \frac{\delta f[\mathbf{x}(t), \mathbf{u}(t), t]}{\delta x} \quad \text{for } x = \hat{x}(k/k) \tag{14}$$

where: $\boldsymbol{Q}$ – state noise covariance matrix;

3) Computing of the filter gain matrix:

$$\mathbf{K}(k+1) = \mathbf{P}(k+1/k)\mathbf{H}^T(k+1) \times \times \begin{bmatrix} \mathbf{H}(k+1)\mathbf{P}(k+1/k)\mathbf{H}^{\mathrm{T}}(k+1) + \\ + \mathbf{R}(k+1) \end{bmatrix}^{-1} \tag{15}$$

where: R – output noise covariance matrix,

$$\mathbf{H}(k+1) = \frac{\delta h[\mathbf{x}(t), t]}{\delta x} \quad \text{for } x = \hat{x}(k+1/k). \tag{16}$$

4) The update of the filter covariance matrix:

$$\mathbf{P}(k+1/k+1) = [\mathbf{I} - \mathbf{K}(k+1)\mathbf{H}(k+1)] \times \times \mathbf{P}(k+1/k) \tag{17}$$

5) The state estimation:

$$\hat{\mathbf{x}}(k+1/k+1) = \hat{\mathbf{x}}(k+1/k) + \mathbf{K}(k+1) \times \times \{\mathbf{y}(k+1) - \mathbf{h}[\hat{x}(k+1/k, k+1)]\} \tag{18}$$

6) Return to step 1.

Some initial parameters are necessary for computing the algorithm of the extended Kalman filter, such as: the state vector in t=0, diagonal noise covariance matrices **Q**[5x5], **R**[2x2], and the filter (predictor) covariance matrix **P**[5x5]. It is assumed, that **w**(k) and **v**(k) are gaussian white-noises, independent, with average values equal zero:
E{w(k)}=0, E{v(k)}=0.

4. SIMULATION RESULTS

Based on the presented algorithm of the extended Kalman filter, different simulation tests of the rotor flux and speed estimation of the induction motor were performed. Parameters of the numerical simulation and tested motor are presented in an Appendix. Numerical calculations were made for mathematical models of the motor and filter in the stationary reference frames and per unit system, using MATLAB. The main problem in the application of the extended Kalman filter algorithm is the suitable choice of elements of noise covariance matrices **Q** and **R.** All these matrices were chosen diagonal, based on the expected covariance of noises. Values of the diagonal matrix **R** elements were assumed equal. Initial parameters of filter covariance matrix **P** were selected by trail and error and presented in the Appendix (Table 1). Simulation were performed for the sinusoidal and for PWM inverter supply and no-load operation of the induction motor.

In the following figures simulation results of the rotor speed, rotor flux and stator current estimation using Kalman filter algorithm are presented, for the sinusoidal supply and nominal motor parameters used in the estimation algorithm.

a)

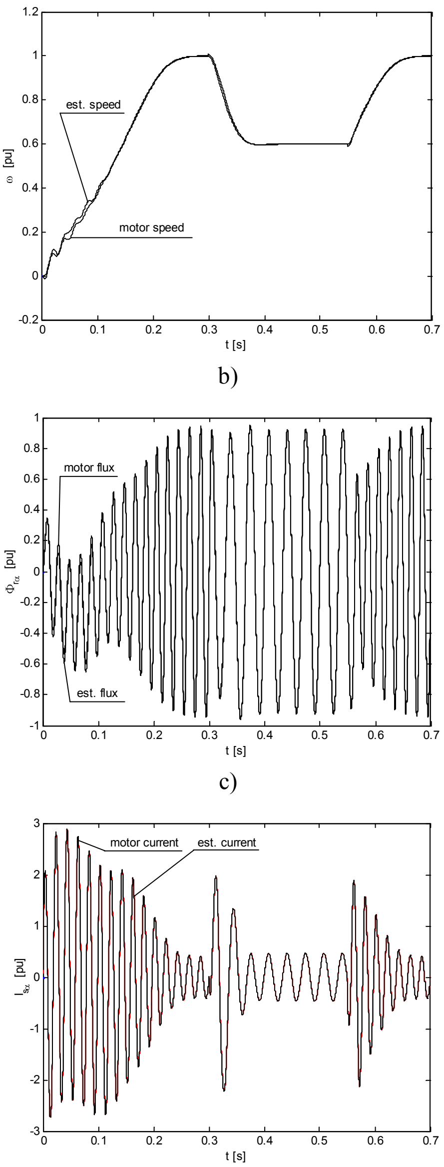


Fig. 1 Actual and estimated state variables of the IM for sinusoidal supply: a) rotor speed, b) rotor flux $\psi_{r\alpha}$, c) stator current $i_{s\alpha}$

In Fig.2 the flux and speed estimation error are presented, defined as follows:

$$\Delta\Psi_{r\alpha} = \Psi_{r\alpha} - \hat{\Psi}_{r\alpha}, \ \Delta\omega = \omega - \hat{\omega} \cdot \tag{19}$$

a)

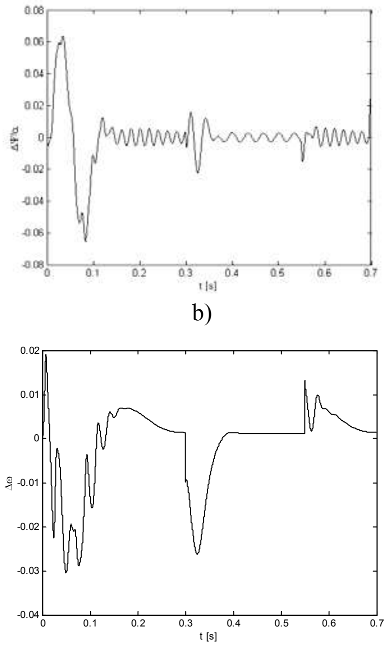


Fig. 2 Transient estimation errors of the rotor flux $\Delta\Psi_{r\alpha}$ (a) and angular speed $\Delta\ \omega$ (b) for sinusoidal supply

In the next figures the similar results are shown in the case of inverter supply of the IM: in Fig.3 – rotor flux and speed estimation errors and in Fig.4 – all transient variables.

a)

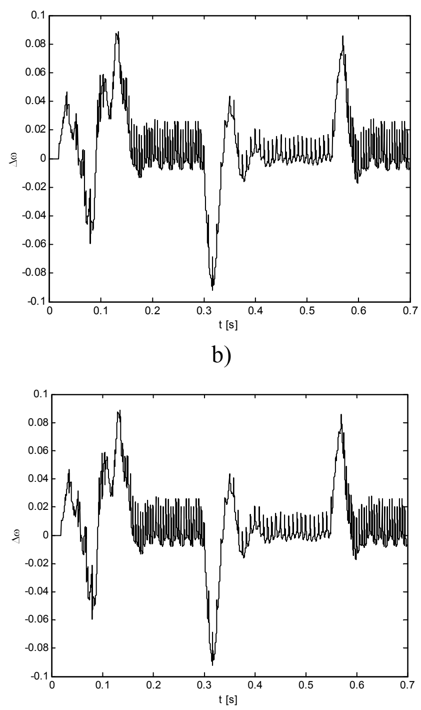
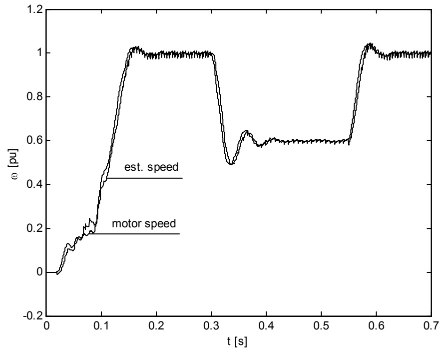


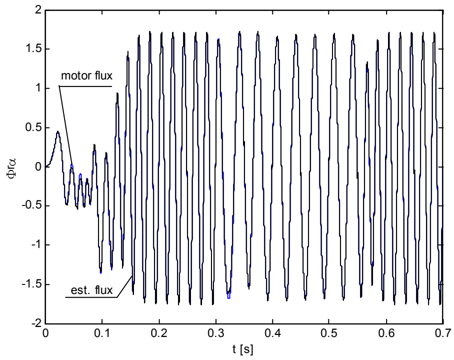
Fig. 3 Transient estimation errors of the rotor flux $\Delta\Psi_{r\alpha}$ (a) and angular speed $\Delta\ \omega$ (b) for PWM supply

The reference frequency was changed during 0.1s from 0 to nominal frequency 50 Hz. After 0.3 s the reference frequency was changed to 30Hz, then at t=0.55s it was again changed to nominal frequency.

a)



b)



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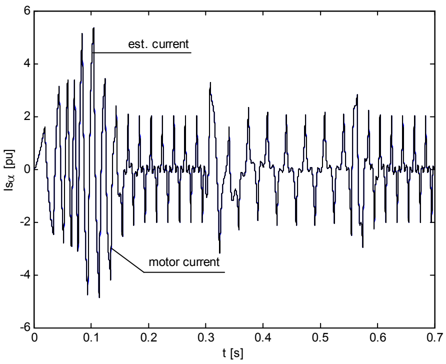


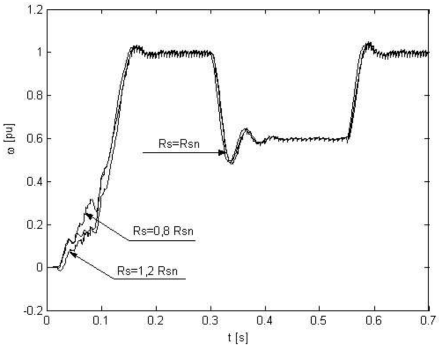
Fig. 4 Actual and estimated state variables of the IM for inverter supply: a) rotor speed, b) rotor flux $\psi_{r\alpha}$, c) stator current $i_{s\alpha}$

In simulation tests the influence of changes of the equivalent circuit parameters and elements of the covariance matrix **Q** on the rotor flux and speed estimation quality was also tested.

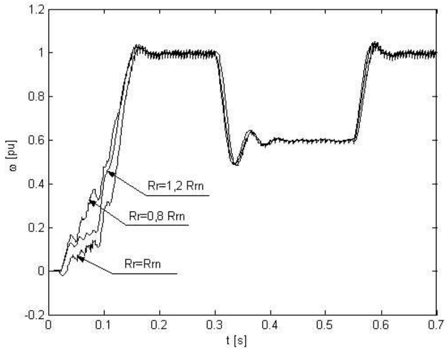
Form the simulation tests performed for nominal motor parameters concludes, that in the case of the sinusoidal supply as well as for PWM supply the rotor flux and speed estimation errors occur only during transient operation of the motor, like starting, change of the speed reference value etc. In the steady-state operation of the motor this error is close to zero for the sinusoidal supply and maximum 2% for the PWM supply.

But in the case when motor parameters used in the Kalman filter algorithm are different than the nominal motor parameters, much bigger transient errors occur in the flux and speed estimates. In Fig.9 the influence of motor parameter changes to the quality of state variables estimation was presented.

a)



b)



c)

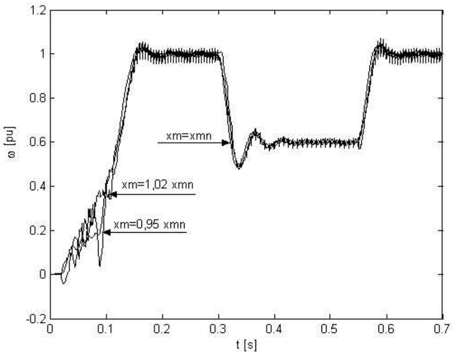


Fig. 5 The influence of motor parameters changes to speed estimation quality: a) stator resistance R_s , b) rotor resistance R_r ,c) magnetising reactance X_M

To have better comparison, the average errors of speed estimation for changing motor parameters were calculated, according to the following formula:

$$Err\,[\%] = \sum_{k=1}^{n} \frac{e_{\omega_k}[\%]}{n}, \tag{20}$$

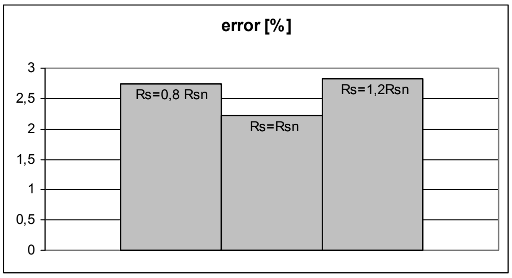
where:

$$e_{\omega_k} = \frac{|\omega_k - \hat{\omega}_k|}{\omega_k} \cdot 100\%, \tag{21}$$

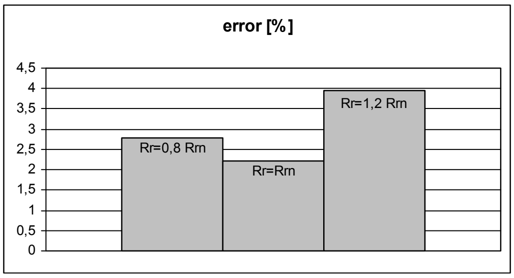
ω - rotor speed, $\hat{\omega}$ - estimated rotor speed,
n – number of all speed samples.

In Fig.6 these average errors calculated for the time period (0.1-0.7) s of the motor operation are presented. The samples from the initial period of the motor starting process were not taken into account due to the relatively big transient estimation error, which had increased significantly the final calculation result in the expression (20).

a)



b)



c)

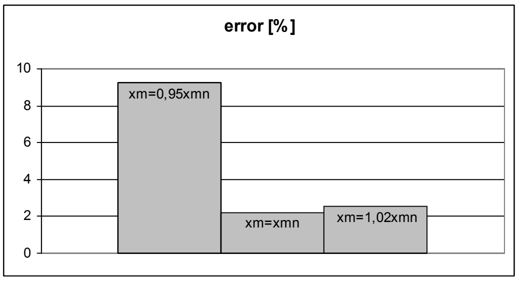


Fig. 6 Average speed estimation errors for motor parameters changes: a) stator resistance R_s , b) rotor resistance R_r , c) magnetising reactance X_M

These average estimation errors, for relatively small changes of motor resistance in the range of ±20% are not significant in the case of motor parameter variations, except the change of the magnetising reactance. But it should be mentioned, that the estimation errors calculated only for the transient process of the motor are much greater and reach the values of 10-12% for stator and rotor resistance R_s , R_r changes and even 35% for the magnetising reactance X_M changes.
The influence of the change of covariance matrix **Q** elements on the motor speed estimation quality was also tested. The speed estimation quality is mostly dependent on **Q**[5,5] element, what was demonstrated in Fig.7. In the next figure the average speed estimation error calculated similarly as in the case of motor parameter changes was presented.

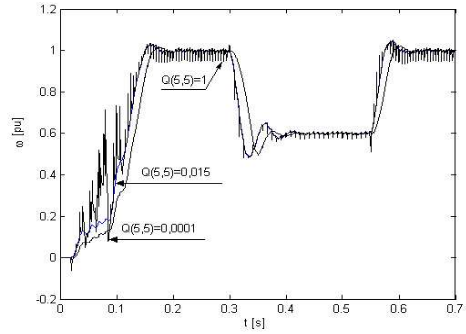


Fig. 7 The influence of Q[5,5] element on the speed estimation quality

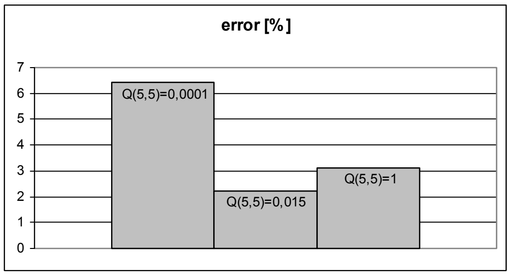


Fig. 8 Average speed estimation error for changes of covariance matrix element Q[5,5]

The average estimation errors calculated only for the transient process of the motor are much greater and reach the values in the range of 15%. So, the speed estimation quality is very much dependent on the suitable choice of the **Q** matrix elements.
The EKF algorithm requires a relatively small integration step used in the numerical algorithm, because of accuracy and stability problems. It is shown in Fig.9, for the integration steps changing in the range of Δt=(0.0001 ÷0.00008)s.

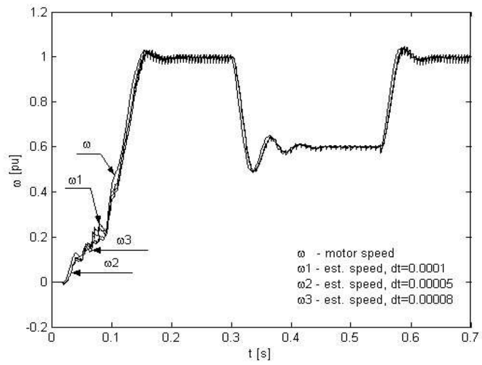


Fig. 9 The influence of the numerical integration step to the stability of EKF algorithm

From the performed simulation tests concludes that the Extended Kalman Filter is not completely robust to the identification errors of the motor parameters. The incorrect value of magnetising reactance used in the EKF algorithm causes significant estimation errors, but if this parameter error is greater than

10%, it can completely destabilise the numerical algorithm of EKF. It is caused by not suitable adaptation of elements of the noise covariance matrix **Q** for the changed motor parameters. If elements of this matrix are not properly chosen, the transient as well as steady-state errors occur in all state variable estimates. It should be mentioned, that elements of this matrix should be adapted to the changes of motor parameters, but it is relatively difficult task and the problem should be solved in a different way.

5. CONCLUSION

The Extended Kalman Filter algorithm is an interesting solution in the task of the IM speed estimation. But its algorithm is much more complicated in comparison with other solutions based on control theory methods [7,8], especially due to significant troubles with the choice of elements of the noise covariance matrix **Q** and **R**. The other problem is the requirement of very small numerical integration step to obtain the accurate solution of difference equations of the EKF algorithm. This requirement is especially important from the practical realisation point of view. Thus only very fast digital signal processor is able to solve the problem of simultaneous estimation of the rotor flux vector and rotor speed in one unified algorithm based on the EKF.

Besides, dynamical properties of the EKF can be controlled only by the choice of covariance matrices **Q** and **R**. In regard to limitations connected with the choice of elements of these matrices, any flexibility does not exist in adjustment of such estimator parameters as the speed of convergence of the rotor flux and the angular speed estimates, dynamics of the estimator response or its robustness to changes of motor parameters on the stage of the filter design. From this point of view, the Luenberger state observers [7,8] seems to a much more flexible solution.

6. APPENDIX

Tab. 1 Simulation parameters

For the sinusoidal supply: sampling step T_s = 0,00008 s, **P** = diag[0.001, 0.001, 0.22, 0.22,0.15], **Q** = diag[0.2, 0.2, 5.0E-2, 5.0E-2, 4.45E-2], **R** = diag[0.01, 0.01]
For the inverter supply: sampling step T_s = 0,00008s, **P** = diag[0.01, 0.01, 0.22, 0.22, 0.15], **Q** = diag[0.4, 0.4, 1.0E-2, 1.0E-2, 1.5E-2], **R** = diag[0.01, 0.01].

Parameters of SDChm-180-M motor:
P_N=5,5kW; U_N=380V; I_N=13,5A; n_N=910 obr/min; η=0,74; cosφ=0,73; M_{max}/M_N=2,1

Tab. 2 Parameters of the equivalent motor circuit

R_s	R_r	X_s	X_r	X_M	[-]
1,085	1,803	32,4	36,329	29,6	[Ω]
0,0665	0,1106	1,9877	1,9877	1,816	[p.u.]

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BIOGRAPHIES

Krzysztof P. Dyrcz, M.Sc., has finished his Master Thesis in 1996 and since this year he is an assistant in the Institute of Electrical Machines, Drives and Measurements at Wroclaw University of Technology. His main field of interest are the state variable estimations methods of the AC motors and microprocessor control techniques of electrical drives.

Teresa Orlowska-Kowalska, Ph.D., D.Sc., prof., has obtained her scientific degrees in 1976, 1990 respectively, at Electrical Engineering Faculty of Wroclaw University of Technology. Since 1993 she has the professor's position at this faculty and is head of Electrical Drives Control Chair in the Institute of Electrical Machines, Drives and Measurements. She is author and co-author of over 100 scientific papers. Her field of interest is the mathematical modelling and microprocessor control of electrical drives and power electronic systems, application of modern control methods to electrical drives, state estimation of AC using observers and neural networks. In the last years her main field of research is connected with neural networks and fuzzy-logic techniques applied to electrical drives control. She is the member of Electrical Engineering Committee of Polish Academy of Science, IEEE, IMACS, EPE, CIGRE and member of the international steering committees of few well-known European conferences.