

## CONTRIBUTION TO RELIABILITY ASSESSMENT OF COMPLEX NETWORKS IN THE POWER SYSTEM

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### SUMMARY

*This paper deals with the analysis and operability assessment of the power system during non-standard operation regimes. Increasing demands on high quality of power delivery and high reliability of the power system put growing pressure on power utilities that have to cope with these matters. The questions of reliability are important in the field of both power generation and power transfer. In order to meet sometimes very tough conditions on operation readiness of the power system, it is necessary to analyze the situation thoroughly. Several methods using different approaches for power system analysis have been worked out. Each of them has its advantages and drawbacks that determine their use in reliability assessment under real operating conditions. Some of these methods are relatively simple but have only limited use due to their lower accuracy. Anyway, they are still usable for some kinds of calculation. On the other hand, much more complex methods are needed in order to achieve higher accuracy of results. But calculations using these methods are usually much longer and more demanding, requiring iterations. Three different methods for analysis of power transfer networks are described in the following text, including evaluation of their limitations.*

**Keywords:** *analysis, reliability, power system, failure, electrical energy, impedance matrix, probability, transmission and distribution of electrical energy, admittance, electrical network*

### 1. INTRODUCTION

Reliability is a relation to consumer. We can characterize it as a general object's property, described as the ability to fulfill required functions, while set operating indicators remain within given boundaries and in time according to technical conditions. In this paper, we present some approaches that are used in power system analysis and thus enable to evaluate system reliability and operation readiness.

When performing power transmission and distribution reliability assessment, it is necessary to choose at first a suitable criterion for operable state. In many cases we are interested only in problems connected with power interruptions, and we assume that individual elements have unlimited power transfer capacity. This is the simplest criterion that premises the operability in every case when the power system graph is formed by one uninterrupted part that contains all consumption nodes. The total generation capacity must be greater or equal to total power load. We do not take into account the power quality in consumption sites, especially the voltage magnitude. When solving complex power systems, we do not regard this criterion as sufficient.

Therefore we next use only those criteria taking into account also the power quality.

### 2. BASIC REQUIREMENTS AND CONCEPT OF CALCULATION

When assessing the fault-free operation probability of the power system we accept the following simplifying assumptions:

- a) We regard a power system as operable if none of its elements is overloaded. The total generation capacity covers the power demand including

power losses, and the voltage at consumption sites is within required limits.

- b) Every failure in the power system causes a transient process. This process is not considered; we concentrate on so-called "steady faults" only, i.e. we try to find out whether the system is able to operate in steady state resulting from the transient process caused by failure.
- c) The degree of risk due to random failures in the power system will be characterized by two major parameters:

Probability of excessive power demand in the system ...  $Q_s$

Mean (expected) value of unsatisfied power demand ...  $Z_s$

The value of  $Q_s$  is expressed in % or relatively ( $Q_s < 1$ ). The complement to 1 is the probability of meeting the power demand  $P_s = 1 - Q_s$ . The value of  $Z_s$  is expressed in MW or, if compared with total power demand, in % or relatively. Both  $Q_s$  and  $Z_s$  values can be calculated not only for the system as a whole but under certain supplement conditions also for a part of the system or for just one consumption node.

Power system reliability assessment is based on the fundamental idea: each element of the power system (e.g. transmission line, generator, bus etc.) can find itself in one of two states:

- "0" state, i.e. state in which the element is disconnected from the system and does not perform its function. This can happen for instance as a result of outage caused by the element's failure or other element's outage resulting from the shutdown due to a planned overhaul or maintenance.
- "1" state, i.e. state in which the element is operating.

If the system is comprised of  $n$  elements, then it can find itself generally in  $2^n$  different states. The state of every single element is given with a probability that can be evaluated statistically from the element's behavior in recent period. Alike, each of  $2^n$  states of the system has a certain probability depending on reliability characteristics of individual elements and on their configuration.

When assessing the power system reliability it is necessary because of high value of number  $2^n$  to carry out analysis of all possible states and choose only those ones that are really probable – this is the first part of the task. The second part lies in evaluation of operability for every state chosen in such a way. This part of the task claims an iterative solution of network (or state in general).

If we know a set of really probable states (together with the value of their probability) in which the power system is not fully operable, then we can work out the probability parameters  $Q_s$ ,  $Z_s$  described above and/or other derived parameters.

Providing reliable power supply while keeping requested power quality in all consumption sites is conditioned by both power generation reliability and power transmission and distribution reliability in power grids. The questions connected with power generation reliability in the power system, e.g. the probability of meeting the total power demand in the power system by a set of power units while considering various influences, are not discussed in this paper.

We concentrate only on questions of power transmission and distribution reliability, while power generating plant in general will be considered as one of the elements with known operating and reliability parameters.

Next, we describe three approaches used for operability assessment of the power system in various states.

### 3. OUTAGE ANALYSIS USING METHOD OF MAXIMUM FLOW IN THE TRANSFER NETWORK

This operability criterion is related to a task of finding the maximum flow in the power transfer network known from graph theory. It is a simplified criterion that does not use iterative solution of the network. But the simplifying assumptions used in its application can be still considered as usable.

The power system diagram is in general comprised of nodes, branches, sources and loads interconnected in certain configuration, e.g. according to Fig. 1. In this figure the values of  $c_1, \dots, c_6$  denote the maximum power transfer capacity of branches (in MVA),  $S_1, S_2, S_3$  denote requested power loads (in MVA) and  $Z_1, Z_2$  power supplies (in MVA) that are available from power sources. The diagram in Fig. 1 can be rearranged into other form depicted in Fig. 2, which contains only one power source  $Z$  of unlimited capacity and only one power load  $S = S_1 + S_2 + S_3$ . The real sources and loads are taken into account as additional branches with power

transfer capacities  $c_7 = Z_1, c_8 = Z_2, c_9 = S_1, c_{10} = S_2, c_{11} = S_3$ .

We work out for diagram according to Fig. 1 so-called maximum flow, i.e. the maximum possible value of power in MVA that can flow from  $Z$  to  $S$ , while none of power transfer capacities  $c_i$  in corresponding branches is exceeded. If this maximum flow is greater or equal to value  $S$ , then the power grid is considered operable, while the opposite case is considered a fault state. The outage of some element or set of elements is represented by the change of value of corresponding power transfer capacity according to Fig. 2 to 0 ( $c_i = 0$ ).

Fig. 2 can be mathematically interpreted as a graph in which the transfer elements  $1 \div 11$  represent graph edges. The solved problem can be then transformed into a task of iterative solution of maximum flow through a given graph with limited capacity of its edges while taking into account various outages. This way we can obtain the answer to question whether or not a given state of the system can transfer requested power load, and thus assess the operational readiness of the power system in this state.

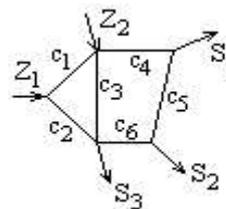


Fig. 1 A graph of an electric power network

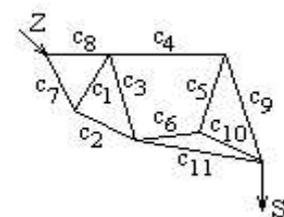


Fig. 2 A rearranged graph from Fig. 1

For finding the maximum flow through a graph with limited capacity of its edges, the Ford-Fulkerson algorithm can be used.

In order to apply a maximum flow method with success, it is necessary to meet some basic assumptions:

1. The transfer capacities of individual elements must be evaluated regarding the maximum allowable voltage drops in individual branches, corresponding e.g. to peak load in the power system. If the condition is met, that each element transfers power lower than the set limit, then no significant voltage drops can occur, which would otherwise result in voltage magnitude in some nodes out of limits. Then it is not necessary to check node voltages when assessing the operability.
2. It is important to assume certain possibilities of voltage regulation in the power system in order to eliminate cases when some of the network parts is overloaded while other is used substantially below its top limit. The network as a whole is then able (in the sense of maximum flow) to transfer requested power load. But in

this case the voltage deviations in individual nodes are significant.

The described method is approximate; in accurate calculations we cannot avoid using the given values of passive parameters of the network (R, L, C). But for quick and rough network reliability assessment it seems to be sufficient.

#### 4. OUTAGE ANALYSIS USING METHOD OF SIMPLIFIED NETWORK SOLUTION (“direct-current mathematical model”)

Basic power equations valid for the  $i^{\text{th}}$  node of the electric network in steady state have this form:

$$\begin{aligned} P_i &= P_{Di} - P_{Oi} = U_i \sum_{k=1}^N U_k A_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) \\ Q_i &= Q_{Di} - Q_{Oi} = U_i \sum_{k=1}^N U_k A_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) \\ i &= 1 \div N \end{aligned} \quad (4.1)$$

where N is a number of network nodes; they are interconnected by branches, each of the branches represents one network element,

$\bar{A}_{ik} = A_{ik} | \alpha_{ik}$ ,  $i, k = 1 \div N$  are so called “calculation” nodal admittances; for  $i \neq k$ ,  $\bar{A}_{ik}$  equals the real admittance between nodes  $i, k$  with the opposite sign; for  $i = k$ ,  $\bar{A}_{ii}$  equals the sum of all admittances (both direct-axis and quadrature-axis) connected to node  $i$ ,

$\bar{U}_i = U_i | \theta_i$  is the complex voltage in node  $i$ ; its magnitude equals phase-to-phase voltage in the node,  $\theta_i$  denotes the phase angle between this voltage and the voltage in so-called reference node  $s$ , where  $\theta_s = 0$ .

$P_{Di}, P_{Oi}$  is the true power supplied and consumed in node  $i$ ,

$Q_{Di}, Q_{Oi}$  is the reactive power supplied and consumed in node  $i$ ; the inductive reactive power is positive.

Next, we accept the following simplifying assumptions:

- a) We neglect the resistance R and leakage G of electric network elements, thus:

$$\bar{A}_{ij} = jB_{ij} = B_{ij} \left| \frac{\pi}{2} \right.$$

- b) The voltages in individual nodes are approximately constant so it is not necessary to take into account the equations for reactive power

- c) We choose node 1 as a reference node, i.e.  $\theta_1 = 0$ , which means that we can eliminate the first one of equations for true power

- d) Assuming the angle  $(\theta_i - \theta_k)$  small enough, we can use the following substitution:  $(\theta_i - \theta_k)$  instead of

$$\cos(\theta_i - \theta_k - \frac{\pi}{2}) = \sin(\theta_i - \theta_k)$$

Then equations (4.1) obtain this form:

$$\begin{aligned} P_i &= P_{Di} - P_{Oi} = \sum_{k=1}^N U_i U_k B_{ik} (\theta_i - \theta_k) = \sum_{k=2}^N R_{ik} \theta_{ik} \\ i &= 2 \div N \end{aligned} \quad (4.2)$$

$$\begin{aligned} \text{where } i \neq k: & \quad R_{ik} = -U_i U_k B_{ik} \\ i = k: & \quad R_{ii} = \sum_{\substack{k=1 \\ k \neq i}}^N U_i U_k B_{ik} \end{aligned}$$

The formula (4.2) can be expressed in matrix notation:

$$\mathbf{P} = \mathbf{P}_D - \mathbf{P}_O = \mathbf{R} \cdot \boldsymbol{\theta} \quad (4.3)$$

It represents so called “direct-current” mathematical model of the power system in steady state. If we know both supplied and consumed true power in nodes ( $P_D, P_O$ ) and passive network parameters, represented in this case by matrix R of order N-1, then by solution of (4.3) the values of  $\theta_2 \div \theta_N$  for N-1 unknown nodes can be found ( $\theta_1 = 0$ ). True power flowing through any branch connecting nodes  $i, k$  is then given by the following equation:

$$P_{ik} = -R_{ik} (\theta_i - \theta_k) \quad (4.4)$$

If power outage in some site occurs (e.g. as a result of failure), then the value of  $P_{Di}$  in this site changes, which leads according to

$$\boldsymbol{\theta} = \mathbf{R}^{-1} (\mathbf{P}_D - \mathbf{P}_O) \quad (4.5)$$

to a change of elements of vector  $\boldsymbol{\theta}$  and thus according to (4.4) to changes in power flows in all branches of the network. If any of the branch currents exceeds the allowable limit the network cannot be considered operable.

If an outage of one or more branches (e.g. transmission line, transformer, bus) occurs (as a result of failure in the power system) while power supply and power load in nodes unchanged, then this shows in equations (4.3) as a change in both matrix R and  $\boldsymbol{\theta}$ .

Before failure stands:

$$\mathbf{P}_{(o)} = \mathbf{R}_{(o)} \cdot \boldsymbol{\theta}_{(o)} \quad (4.6)$$

After failure stands:

$$\mathbf{P}_{(o)} = (\mathbf{R}_{(o)} + \Delta\mathbf{R})(\boldsymbol{\theta}_{(o)} + \Delta\boldsymbol{\theta}) \quad (4.7)$$

Neglecting the term  $\Delta\mathbf{R} \cdot \Delta\boldsymbol{\theta}$  (of second order and therefore of low magnitude) and using (4.6) and (4.7) we get:

$$\Delta\boldsymbol{\theta} = -\mathbf{R}_{(o)}^{-1} \cdot \Delta\mathbf{R} \cdot \boldsymbol{\theta}_{(o)} \quad (4.8)$$

Similarly from (4.4):

$$\Delta P_{ik} = -R_{ik}(\Delta\theta_i - \Delta\theta_k) \quad (4.9)$$

Equation (4.8) reflects the changes of voltage phase angles while equation (4.9) reflects the changes of power flows in branches, both resulting from alterations in the power system configuration.

When assessing operational readiness of the power system in various states, we know  $\mathbf{R}^{-1}_{(o)}$ ,  $\boldsymbol{\theta}_{(o)}$ , and/or  $P^{(o)}_{ik}$  representing the original fault-free state. It is necessary to work out matrix  $\Delta\mathbf{R}$  for every possible fault state.

From the described derivation results the fact that if we remove one branch in the power system, e.g. between nodes p, q, then only elements  $\Delta R_{pp}$ ,  $\Delta R_{pq}$ ,  $\Delta R_{qp}$  and  $\Delta R_{qq}$  of matrix  $\Delta\mathbf{R}$  will be non-zero:

$$\Delta R_{pp} = \Delta R_{qq} = -\Delta R_{pq} = -\Delta R_{qp} = R_{pq} \quad (4.10)$$

By substituting into (4.8) we obtain:

$$\Delta\boldsymbol{\theta}^{(p,q)} = -\mathbf{R}_{(o)}^{-1}(\boldsymbol{\theta}_{(o)p} \cdot \Delta\mathbf{R}_p - \boldsymbol{\theta}_{(o)q} \cdot \Delta\mathbf{R}_q) \quad (4.11)$$

$$\Delta P_{ik}^{(p,q)} = -R_{ik}(\Delta\theta_i^{(p,q)} - \Delta\theta_k^{(p,q)}) \quad (4.12)$$

for  $ik \neq pq$

In equations (4.11) and (4.12) denotes:

$\Delta\boldsymbol{\theta}^{(p,q)}$  changes of voltage angles in nodes  $i = 2 \div N$  when branch p, q removed

$\Delta P_{ik}^{(p,q)}$  change of power flow in branch i,  $k \neq p, q$  when branch p, q removed

$\boldsymbol{\theta}_{(o)p}$ ,  $\boldsymbol{\theta}_{(o)q}$  voltage angles in nodes p, q before removing branch p, q

$\Delta\mathbf{R}_p$ ,  $\Delta\mathbf{R}_q$  the p<sup>th</sup> and q<sup>th</sup> column of matrix  $\Delta\mathbf{R}$ .

The method enables to find out relatively easily whether or not a branch of the power system is overloaded during the outage of some of the network elements (source, transmission line, transformer) with no need to iterate the calculation. It can be easily generalized for case of multiple outages (e.g. bus) because the dependence of voltage angle changes  $\Delta\boldsymbol{\theta}$  on network configuration changes  $\Delta\mathbf{R}$  is according to (4.8) linear.

Regarding the accepted assumptions, the method is not very accurate so it should be used only for quick and rough reliability calculations. The

approach using the nodal impedance matrix is needed for more accurate outage analysis.

## 5. OUTAGE ANALYSIS FOR VARIOUS ELEMENTS OF THE POWER SYSTEM USING THE METHOD OF NODAL IMPEDANCE MATRIX

Nodal impedance matrix  $\mathbf{Z}$  of the power system expresses relationship between voltages in nodes of the power system ( $U_1, U_2, \dots, U_N$ ) and node currents ( $I_1, I_2, \dots, I_N$ ) according to equation (5.1):

$$\mathbf{U} = \mathbf{Z} \cdot \mathbf{I} \quad (5.1)$$

It is the inverse to so-called nodal admittance matrix  $\mathbf{A}$  with elements  $A_{ik}$ ,  $A_{ji}$ . There have been many algorithms worked out for its construction.

Suppose the power system has been solved for one particular basic state denoted with (0) index. Thus we know all node voltages  $U_i^{(0)}$ ,  $i = 1 \div N$  and branch currents  $I_{ik}^{(0)}$ ,  $ik = 1 \div M$  for this state. This state is also acceptable in terms of operability conditions.

Suppose furthermore the nodal impedance matrix  $\mathbf{Z}^{(0)}$  is known for the whole network, including both branch impedances of the power system and equivalent impedances of individual loads:

$$Z_{Si} = \frac{|U_i|^2}{P_{Si} - jQ_{Si}} \quad (5.2)$$

where  $P_{Si}$  and  $Q_{Si}$  are the true and reactive power consumed in node i,  $|U_i|$  is the magnitude of phase-to-phase voltage in node i. If  $Q_{Si} > 0$  then the reactive power is inductive, and on the contrary if  $Q_{Si} < 0$  then the reactive power is capacitive.

Power supply in nodes of the power system will be represented by node currents  $I_i$ ,  $i = 1 \div N$ . For the original (basic) state stands:

$$\mathbf{U}^{(0)} = \mathbf{Z}^{(0)} \cdot \mathbf{I}^{(0)} \quad (5.3)$$

where  $\mathbf{U}^{(0)}$  is a vector of node voltages  $U_i^{(0)}$ ,  $i = 1 \div N$  and  $\mathbf{I}^{(0)}$  vector of nodal currents  $I_i^{(0)}$ ,  $i = 1 \div N$ . If in this state a fault occurs, e.g. branch switch-off or source outage, then it shows in a change of node voltages  $U_i^{(0)}$  by value of  $\Delta U_i$  and change of branch currents  $I_{ik}^{(0)}$  by value of  $\Delta I_{ik}$ . If any operating quantity exceeds allowable limits this new state will be considered a fault state (non-operable) in terms of power system reliability assessment. We further adopt the following simplifications.

A network will be considered operable if each branch is loaded below the transfer capacity, which is scheduled for every single branch in advance respecting voltage drops. Then there will be no significant voltage drops that would otherwise put voltage in some parts of the power system out of allowable limits. Thus no further check of voltage magnitude in individual nodes will be needed.

Let's derive a formula for the current changes in individual branches  $\Delta I_{ik}$ ,  $(ik) = 1 \div M$  during fault outage of one branch –  $(mn)$ . Before the outage, the current  $I_{mn}^{(0)}$  flowed through the branch. Suppose further a network without zero-valued supply currents  $I_i^{(0)}$  in nodes  $i = 1 \div N$ . Should the current  $I_{mn}^{(0)}$  flow through branch  $(mn)$ , it is necessary that node  $m$  is supplied with current  $I_m$ ; for this stand the following equations:

$$\begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_N \end{bmatrix} = \begin{bmatrix} Z_{11}^{(0)} & Z_{12}^{(0)} & \cdots & Z_{1N}^{(0)} \\ Z_{21}^{(0)} & Z_{22}^{(0)} & & Z_{2N}^{(0)} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1}^{(0)} & Z_{N2}^{(0)} & \cdots & Z_{NN}^{(0)} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_m \\ \vdots \\ 0 \end{bmatrix} \quad (5.4)$$

$$\frac{U_m - U_n}{z_{mn}} = I_{mn}^{(0)} \quad (5.5)$$

where  $z_{mn}$  is the direct-axis impedance of branch  $(mn)$ . From (5.4):

$$U_m = Z_{mm}^{(0)} \cdot I_m \quad (5.6a)$$

$$U_n = Z_{nn}^{(0)} \cdot I_m \quad (5.6b)$$

By substituting (5.6a,b) into (5.5) and rearranging we get:

$$I_m = \frac{z_{mn}}{Z_{mm}^{(0)} - Z_{nn}^{(0)}} \cdot I_{mn}^{(0)} \quad (5.7)$$

Currents in all remaining branches resulting from supply current  $I_m$  into node  $m$  according to (5.5) and using (5.6a,b) are worked out as follows:

$$I_{ik}^{(0)} = \frac{U_i^{(0)} - U_k^{(0)}}{z_{ik}} = \frac{Z_{im}^{(0)} - Z_{km}^{(0)}}{z_{ik}} \cdot I_m \quad (5.8)$$

If we eliminate now branch  $(mn)$ , the nodal impedance matrix will change into  $Z^{(1)}$  and assuming the same injected current  $I_m$  according to (5.7), the magnitude of currents in other network branches  $(ik)$  can be found as follows:

$$I_{ik}^{(1)} = \frac{Z_{im}^{(1)} - Z_{km}^{(1)}}{z_{ik}} \cdot I_m \quad (5.9)$$

It's obvious from (5.9) that  $I_{mn}^{(1)} = 0$ , the branch  $(mn)$  was removed. The branch currents have changed by this value:

$$\Delta I_{ik} = I_{ik}^{(1)} - I_{ik}^{(0)} \quad (5.10)$$

By substituting (5.7), (5.8) and (5.9) into (5.10) we obtain:

$$\Delta I_{ik} = I_{mn}^{(0)} \cdot \frac{z_{mn}}{z_{ik}} \cdot \frac{(Z_{im}^{(1)} - Z_{km}^{(1)}) - (Z_{im}^{(0)} - Z_{km}^{(0)})}{Z_{mm}^{(0)} - Z_{nn}^{(0)}} \quad (5.11)$$

The branch current changes  $\Delta I_{ik}$  against the original currents  $I_{ik}^{(0)}$  enable us to assess the power system operation readiness in terms of branch overload.

When it is necessary to assess the multiple outages of two or more branches, the same method can be applied. For each afflicted branch must be worked out the corresponding current supplied into one of the two branch's nodes. Matrix  $Z^{(1)}$  is then a matrix corresponding to the resulting network after elimination of the afflicted branches.

The major problem in application of this method is finding a new impedance matrix  $Z^{(1)}$  when assessing individual outages. Without its derivation, we describe here the algorithm for construction of matrix  $Z$  by gradual adding of branches, which can be used even when changing matrix  $Z^{(1)}$  into  $Z^{(0)}$ . Eliminating a branch of impedance  $z_{mn}$  can be regarded as adding a parallel branch of impedance  $-z_{mn}$ , so the same formulas as in the described algorithm can be used.

The construction of nodal impedance matrix proceeds according to the following procedure. If the network contains  $N + 1$  nodes marked  $0, 1, 2, \dots, N$ , where node  $0$  is the reference node (voltages of other nodes are related to it), then the resulting matrix is of order  $N$ . The nodal impedance matrix for a circuit, containing just one branch between nodes  $0$  and  $1$  of impedance  $z_{10}$ , is of first order and has this form:

$$\mathbf{Z} = z_{10} \quad (5.12)$$

Suppose now we know the nodal impedance matrix  $\mathbf{Z}$  for a partial circuit consisting of  $n$  nodes and reference node  $0$ :

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix} \quad (5.13)$$

Adding a new branch  $(pq)$  of impedance  $z_{pq}$  can result in two possibilities:

1. A new node  $q$  arises. In this case a new matrix  $\mathbf{Z}'$  will have one additional column and one additional row in comparison with matrix  $\mathbf{Z}$  from (5.13) corresponding with the new node  $q$ . The other elements of the matrix remain the same. For elements of the  $q^{\text{th}}$  row and column stands:

$$\begin{aligned} Z_{qi} &= Z_{pi}, & Z_{iq} &= Z_{ip}, & i &= 1 \div n, i \neq q \\ Z_{qq} &= Z_{pp} + z_{pq} \end{aligned} \quad (5.14)$$

If  $p$  is the reference node ( $p = 0$ ) equations (5.14) are also valid, and

$$Z_{pi} = 0, \quad i = 1 \div n, \quad i \neq q \quad (5.15)$$

2. No new node arises. In this case the original matrix  $Z$  retains its dimensions but the values of all the elements change. The elements of a new matrix  $Z'$  are given by:

$$Z'_{ij} = Z_{ij} - \frac{(Z_{pi} - Z_{qi})(Z_{pj} - Z_{qj})}{Z_{pp} + Z_{qq} + z_{pq}}, \quad i, j = 1 \div n \quad (5.16)$$

If  $p$  is the reference node ( $p = 0$ ) equation (5.16) is also valid, whereas

$$Z_{pi} = Z_{pj} = Z_{pp} = 0, \quad i, j = 1 \div n \quad (5.17)$$

Even if the described method can be generalized for case when network branches are mutually bound, the simple equations (5.14) ÷ (5.17) are sufficient in real conditions. If for instance several transmission lines are attached to the same power pylons with existing coupling, they can be replaced by one or more branches with self impedance that respect the mutual influence of power lines.

Now let's get back again to outage analysis of the power system branches. The outage of branch (mn) of impedance  $z_{mn}$  can be regarded as an addition of branch (mn) of impedance  $-z_{mn}$  without creation of a new node. According to (5.16) for elements of matrix  $Z^{(1)}$  stands:

$$Z^{(1)}_{ij} = Z^{(0)}_{ij} - \frac{(Z^{(0)}_{mi} - Z^{(0)}_{ni})(Z^{(0)}_{mj} - Z^{(0)}_{nj})}{Z^{(0)}_{mm} + Z^{(0)}_{nn} - z_{mn}} \quad (5.18)$$

According to this formula it is possible to find the impedances  $Z^{(1)}_{im}$ ,  $Z^{(1)}_{km}$ :

$$Z^{(1)}_{im} = Z^{(0)}_{im} - \frac{(Z^{(0)}_{mi} - Z^{(0)}_{ni})(Z^{(0)}_{mm} - Z^{(0)}_{nm})}{Z^{(0)}_{mm} + Z^{(0)}_{nn} - z_{mn}} \quad (5.19)$$

$$Z^{(1)}_{km} = Z^{(0)}_{km} - \frac{(Z^{(0)}_{mk} - Z^{(0)}_{nk})(Z^{(0)}_{mm} - Z^{(0)}_{nm})}{Z^{(0)}_{mm} + Z^{(0)}_{nn} - z_{mn}} \quad (5.20)$$

Then we can calculate the value of  $\Delta I_{ik}$ . We substitute (5.19) and (5.20) into (5.11). The result is as follows:

$$\Delta I_{ik} = I^{(0)}_{mn} \frac{z_{mn}}{z_{ik}} \frac{(Z^{(0)}_{mk} - Z^{(0)}_{nk}) - (Z^{(0)}_{mi} - Z^{(0)}_{ni})}{Z^{(0)}_{mm} + Z^{(0)}_{nn} - z_{mn}} \quad (5.21)$$

Formula (5.21) is final and shows how much the currents in branches (ik) of impedance  $z_{ik}$  change when an outage of branch (mn) of impedance  $z_{mn}$  occurs, if the pre-outage current of this branch was  $I^{(0)}_{mn}$ . It is necessary for the calculation of the current change  $\Delta I_{ik}$  to know the elements of the original nodal impedance matrix  $Z^{(0)}$ .

So far the outages of certain branches of the power system have been analyzed. Much easier task

is the analysis of power source outage in certain node of the power system. In this case the first step is to find out whether or not the remaining power sources are able to meet the actual power demand. If not, this state of the power system must be considered a fault state.

But if the remaining power sources are able to meet power demand, then we have to further check whether some of the branches will not become overloaded. It can be done using matrix  $Z$ . This case is easier in comparison with branch outage because this time the original nodal impedance matrix  $Z^{(0)}$  does not change. In equation (5.3) only the nodal current vector  $I^{(0)}$  changes into  $I^{(1)}$  in such a way that  $I_p^{(1)} = 0$  ( $p$  is index of the node where source outage occurs). The current changes in individual branches  $\Delta I_{ik}$  are then using (5.5) and (5.10) given by:

$$\Delta I_{ik} = I^{(1)}_{ik} - I^{(0)}_{ik} = \frac{(U_i^{(1)} - U_k^{(1)}) - (U_i^{(0)} - U_k^{(0)})}{z_{ik}} \quad (5.22)$$

According to (5.3) and (5.6a,b) we can write:

$$U_i^{(0)} = \sum_{j=1}^N Z^{(0)}_{ij} \cdot I_j^{(0)} \quad (5.23a)$$

$$U_k^{(0)} = \sum_{j=1}^N Z^{(0)}_{kj} \cdot I_j^{(0)} \quad (5.23b)$$

$$U_i^{(1)} = \sum_{j=1}^N Z^{(0)}_{ij} \cdot I_j^{(1)} = U_i^{(0)} - Z^{(0)}_{ip} \cdot I_p^{(0)} \quad (5.24a)$$

$$U_k^{(1)} = \sum_{j=1}^N Z^{(0)}_{kj} \cdot I_j^{(1)} = U_k^{(0)} - Z^{(0)}_{kp} \cdot I_p^{(0)} \quad (5.24b)$$

Hence after substituting into (5.22) we get:

$$\Delta I_{ik} = \frac{Z^{(0)}_{kp} - Z^{(0)}_{ip}}{z_{ik}} I_p^{(0)} \quad (5.25)$$

Equations (5.22), (5.25) express how much the currents in the power system change and what is the effect on the power flows in individual branches during an outage of power source or branch in the power system, while power demand remains the same. Based on comparison with the scheduled power flows that respect the voltage constraints, it can be assessed whether or not the state of the power system is acceptable.

## 6. CONCLUSION

Outage analysis using method of maximum flow in the transfer network is approximate; in accurate calculations we cannot avoid using the passive parameters of network ( $R$ ,  $L$ ,  $C$ ). But for rough network reliability assessment it seems to be sufficient.

Regarding the accepted assumptions, the outage analysis using method of simplified network solution is not very accurate, so it should be used only for quick and rough reliability calculations. The approach using the nodal impedance matrix is needed for more accurate outage analysis.

Outage analysis for various elements of the power system using the method of nodal impedance matrix seems to be optimum in the sense of accuracy. Based on comparison with the scheduled power flows that respect the voltage constraints, using the formulas described in the paper, it can be assessed whether or not the state of the power system is acceptable.

*This paper was written as a part of the GAČR project (GAČR 102/02/0949*

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