MODELLING OF A LINEAR INDUCTION MOTOR IN DYNAMICAL PERFORMANCE

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SUMMARY

This paper deals with modelling of the linear induction motor (LIM). Linear induction motors fall into the category of special electrical machines in which the electric energy is converted directly into mechanical energy of translatory motion. LIMs are the most popular ones from the group of linear electric motors, which consists also of DC motors, synchronous motors, reluctance motors, stepping motors, oscillating and hybrid motors.

In the paper mathematical model of a tubular LIM is described by a set of differential equations, which are best solved by numerical methods. Knowledge of the system behaviour under different operating conditions is necessary for proper selection of a LIM for an electrical drive system. Both electromagnetic and mechanical steady state and transient characteristics are important. In many cases, general information about how electrical parameters of the drive (including the control circuit, inertia, friction, and stiffness of the system) influence the transients is necessary, and methods of modelling and digital simulation are the most useful. Nowadays, digital simulation is predominant.

In the case of digital simulation, the mathematical model presents a digital system that is programmed on the basis of mathematical equations describing a real tubular linear induction motor. Finally the calculated values are compared with those measured on a real LIM.

Keywords: *linear tubular induction motor, modelling, dynamic mathematical model, state space*

1. INTRODUCTION

Linear induction motors fall into the group of that special-purpose electrical drives that transform electric energy directly to the translating motion mechanical energy.

Geometrically is the traditional asynchronous motor arranged in such a way that the rotating circular magnetic field move along circumference of the air gap in synchronous speed, while the electromagnetic forces generated between electrical circuits on the stator bring about the rotor rotary movement.

However, the very same magnetic field and forces may elicit linear movement too. Geometric arrangement of a flat linear motor with single-sided arrangement can be imagined so that the traditional induction motor primary part would be cut up in halves along the radial plane, and then it would be uncoiled. Magnetic field of in this way arranged motor would move gradually in the air gap along the primary part and due to the electromagnetic force the machine secondary part would move in straight direction.

The LIM can be designed either as flat one-sided devices (with short secondary or short primary part, respectively) or as two-sided ones. One of other possible arrangements of the linear induction motor is the one with cylindrical air gap (so called tubular motor), which induced linear motion in the cylinder axis direction. Such a motor would be arrived at if the flat one-sided linear motor would be coiled into a cylinder around the axis running parallel with the magnetic field movement direction.

LIMs are utilized to induce force (most frequently, actuators run at insignificant speeds), to induce energy (boosters, e.g. at lift-off of the aircraft) or to induce high powers and speeds (in transportation systems). The LIM are extensively utilized within transportation systems, ranging from small cargo drives (used at airports, exhibitions, elevators, etc.) to transportation of bulky loads. Of significant importance are also in manufacturing processes (power hammers, mills, presses, textile machines, robotics, etc.). A highly important domain of their utilization is also the field of industrial testing and research (simulation of car collisions, airplane and car models excessive accelerations in wind tunnels, etc.) The most important LIM application prospective is seen in the high-speed transportation system based on the magnetic air pillow hovering principle.

2. THE TUBULAR LIM MATH MODEL

The tubular linear asynchronous motor, intended for inducing force – the actuator, will be considered in the following. At deriving the linear motor mathematical model we took the following assumptions to be true:

- primary windings are symmetrical
- only the fundamental harmonic of MMF exists
- there is no neutral wire
- the LIM is connected to an infinite bus
- there are no slots and the air-gap is uniform
- there are none longitudinal end effects
- the magnetic circuit is unsaturated

The machine mathematical model can be, both in the steady and the transient states, expressed by a system of non-linear differential equations.
For expressing the motor mathematical model two approaches were used:

- mathematical description expressed by use of spatial vectors (physically more figurative approach)
- the equations state notation (more general approach)

2.1 The linear induction motor equations

Made use of at creating the LIM mathematical model was one of the alternatives of how to simplify model of the motor – i.e. transformation of 3-phase to an equivalent 2-phase system, by which number of equations to be solved decreased.

Based on the analogy between the linear induction motor and rotary induction motors the well-known equations, in the rotating system x, y, for rotary movements can be, using the below equations:

$$\omega = \frac{\pi}{\tau} v \qquad \text{and} \qquad r = \frac{\tau}{\pi} \,, \tag{1}$$

rewritten to equations holding for the tubular linear induction motor:

$$u_{1x} = R_1 i_{1x} + \frac{d\Psi_{1x}}{dt} - \frac{\pi}{\tau} v_1 \Psi_{1y} \,, \tag{2}$$

$$u_{1y} = R_1 i_{1y} + \frac{d\Psi_{1y}}{dt} + \frac{\pi}{\tau} v_1 \Psi_{1x} \,, \tag{3}$$

$$0 = R_2 i_{2x} + \frac{d\Psi_{2x}}{dt} - \frac{\pi}{\tau}(v_1 - v)\Psi_{2y} \,, \tag{4}$$

$$0 = R_2 i_{2y} + \frac{d\Psi_{2y}}{dt} + \frac{\pi}{\tau}(v_1 - v)\Psi_{2x} \,. \tag{5}$$

The motor internal force:

$$F = \frac{3\pi}{2\tau}\left(\Psi_{1x} i_{1y} - \Psi_{1y} i_{1x}\right). \tag{6}$$

The relation expresses the movement equation:

$$\frac{dv}{dt} = \frac{1}{m}\left(F - F_z\right). \tag{7}$$

The primary linkage fluxes:

$$\Psi_{1x} = L_1 i_{1x} + L_h i_{2x} \,, \tag{8}$$

$$\Psi_{1y} = L_1 i_{1y} + L_h i_{2y} \,. \tag{9}$$

The secondary linkage fluxes:

$$\Psi_{2x} = L_2 i_{2x} + L_h i_{1x} \,, \tag{10}$$

$$\Psi_{2y} = L_2 i_{2y} + L_h i_{1y} \,. \tag{11}$$

Rendered from equations (8) to (11) are currents of both the primary and secondary parts, while for the sake of their simpler form introduced was the overall leakage factor $\sigma = 1 - \frac{L_h^2}{L_1 L_2}$:

$$i_{1x} = \frac{1}{\sigma L_1}\Psi_{1x} - \frac{L_h}{\sigma L_1 L_2}\Psi_{2x} \,, \tag{12}$$

$$i_{1y} = \frac{1}{\sigma L_1}\Psi_{1y} - \frac{L_h}{\sigma L_1 L_2}\Psi_{2y} \,, \tag{13}$$

$$i_{2x} = \frac{1}{\sigma L_2}\Psi_{2x} - \frac{L_h}{\sigma L_1 L_2}\Psi_{1x} \,, \tag{14}$$

$$i_{2y} = \frac{1}{\sigma L_2}\Psi_{2y} - \frac{L_h}{\sigma L_1 L_2}\Psi_{1y} \,. \tag{15}$$

Following adaptation and subsequent installing of equations (12) to (15) into relations expressed by equations (2) to (5) obtained have been the equations (16) to (19):

$$\frac{d\Psi_{1x}}{dt} = u_{1x} - R_1\frac{1}{\sigma L_1}\Psi_{1x} + R_1\frac{L_h}{\sigma L_1 L_2}\Psi_{2x} + \frac{\pi}{\tau}v_1\Psi_{1y} \tag{16}$$

$$\frac{d\Psi_{1y}}{dt} = u_{1y} - R_1\frac{1}{\sigma L_1}\Psi_{1y} + R_1\frac{L_h}{\sigma L_1 L_2}\Psi_{2y} - \frac{\pi}{\tau}v_1\Psi_{1x} \tag{17}$$

$$\frac{d\Psi_{2x}}{dt} = -R_2\frac{1}{\sigma L_2}\Psi_{2x} + R_2\frac{L_h}{\sigma L_1 L_2}\Psi_{1x} + \frac{\pi}{\tau}(v_1 - v)\Psi_{2y} \tag{18}$$

$$\frac{d\Psi_{2y}}{dt} = -R_2\frac{1}{\sigma L_2}\Psi_{2y} + R_2\frac{L_h}{\sigma L_1 L_2}\Psi_{1y} - \frac{\pi}{\tau}(v_1 - v)\Psi_{2x} \tag{19}$$

which completed with equations (6) and (7) represent the linear motor flux mathematical model, where:

F, F_L	-	force and load force
i_{1x}, i_{1y}, i_{2x}, i_{2y}	-	*x, y* components of stator and rotor current
L_1, L_2, L_h	-	stator, rotor and main inductance
m	-	mass
R_1, R_2	-	stator and rotor winding resistance
u_{1x}, u_{1y}	-	*x, y* components of stator voltage
v_1, v	-	synchronous and mechanical velocity
σ	-	leakage factor
τ	-	pole pitch
Ψ_{1x}, Ψ_{1y}, Ψ_{2x}, Ψ_{2y}	-	*x, y* components of stator and rotor magnetic fluxes
ω	-	angular frequency

where subscript 1 belongs to stator variables and subscript 2 belongs to rotor variables.

2.2 The state model of the LIM

Whereas the linear motor mathematical model is at disposal in the form of a system of differential and algebraic equations derived from the description can be, by appropriate introduction of state variables, state equations. For the state quantities it is highly convenient to select significant drive variables such as currents, magnetic fluxes, the speed, trajectory, etc.
The general state equation for the ac machines can be written as:

$$\frac{d}{dt}\boldsymbol{X(t)} = \boldsymbol{AX(t)} + \boldsymbol{Bu(t)}, \tag{20}$$

where $\boldsymbol{X(t)} = [\boldsymbol{X}_1, \boldsymbol{X}_2]$ is the two-component vector of electromagnetic state variables, $\boldsymbol{u(t)} = [\boldsymbol{u}_1, \boldsymbol{u}_2]$ the input function and $\boldsymbol{A}$ the ac machine matrix whose elements are functions of synchronous speed and machine shaft angular speed and $\boldsymbol{B}$ is input matrix. The vector components $\boldsymbol{X_1}$, $\boldsymbol{X_2}$ may be any two vectors arbitrarily selected from among the linkage flux vectors or currents vectors. The input is always a pair of voltage vectors $\boldsymbol{u_1}$, $\boldsymbol{u_2}$.
Taking as the state variables for the LIM the stator and rotor flux linkage vectors then for a synchronously rotating system of coordinates *x*, *y*, state equation (20) of the LIM can be resolved into two components to obtain:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -\frac{R_1}{\sigma L_1} & \frac{\pi}{\tau}v_1 & \frac{R_1 L_h}{\sigma L_1 L_2} & 0 \\ -\frac{\pi}{\tau}v_1 & -\frac{R_1}{\sigma L_1} & 0 & \frac{R_1 L_h}{\sigma L_1 L_2} \\ \frac{R_2 L_h}{\sigma L_1 L_2} & 0 & -\frac{R_2}{\sigma L_1} & \frac{\pi}{\tau}(v_1 - v) \\ 0 & \frac{R_2 L_h}{\sigma L_1 L_2} & -\frac{\pi}{\tau}(v_1 - v) & -\frac{R_2}{\sigma L_1} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \end{bmatrix} \tag{21}$$

The motion equation was obtained by introducing equations (12), (13) into equation (6) and by subsequent introduction into equation (7).

$$\frac{dv}{dt} = \frac{3\pi}{2m\tau}x_1\left(\frac{1}{\sigma L_1}x_2 - \frac{L_h}{\sigma L_1 L_2}x_4\right) - \dots$$
$$\dots - \frac{3\pi}{2m\tau}x_2\left(\frac{1}{\sigma L_1}x_1 - \frac{L_h}{\sigma L_1 L_2}x_3\right) - \frac{1}{m}F_z \tag{22}$$

The stator current components derivations were obtained by substituting equations (16)-(19) into deriving equations (12) and (13).

$$\dot{i}_{1x} = \frac{1}{\sigma L_1}\left(u_{1x} - R_1 i_{1x} + \frac{\pi}{\tau}v_1 \sigma L_1 i_{1y}\right) + \dots$$
$$\dots + \frac{1}{\sigma L_1}\left[\left(L_h \frac{R_2}{L_2^2}\right)(x_3 - L_h i_{1x}) + \frac{\pi}{\tau}v\frac{L_h}{L_2}x_4\right] \tag{23}$$

$$\dot{i}_{1y} = \frac{1}{\sigma L_1}\left(u_{1y} - R_1 i_{1y} - \frac{\pi}{\tau}v_1 \sigma L_1 i_{1x}\right) + \dots$$
$$\dots + \frac{1}{\sigma L_1}\left[\left(L_h \frac{R_2}{L_2^2}\right)(x_4 - L_h i_{1y}) - \frac{\pi}{\tau}v\frac{L_h}{L_2}x_3\right] \tag{24}$$

3. SIMULATIONS RESULTS

The analytical solution to the system of equations in (16-19) and (6-7) or in (21-24) is very difficult due to non-linearity. The problem is simple when computers and standard library routines are used for solving non-linear differential equations. Thus, digital simulation of a linear induction motor is equivalent to finding a numerical solution to the equations, along with graphical displaying of the results.

Testing of the mathematical model and measurement were provided on a tubular linear induction motor TLM-60 with the following parameters:

U = 3 x 380 V/50 Hz, I_N = 5.6 A, P=3100 W, v = 1.8 m.s^{-1}, $\cos\varphi$ = 0.841, F_N = 200 N, m_1 = 6 kg, R_1 = 15.38Ω, R_2 = 47.6 Ω, $X_{1\sigma} = X_{2\sigma}$ = 14.9225Ω, X_h = 33.3333Ω, v_1 = 3.675 m.s^{-1}, τ = 0.036 m, m_2 = 1.1 kg

Parameters of the tubular linear induction motor needed for solving the system of the equations was found experimentally at *v=0* and are assumed constant despite of their variation with slip.

The modelled waveforms of: the stator current components, stator magnetic flux components and the rotor linkage magnetic flux for the case of motor no-load start-up, at rated loading: *t* = 0,25s – 100% F_N, as well as at altered loading: *t* = 0,45s to 75% F_N are shown in figures below (Fig. 1, Fig. 2, Fig. 3).

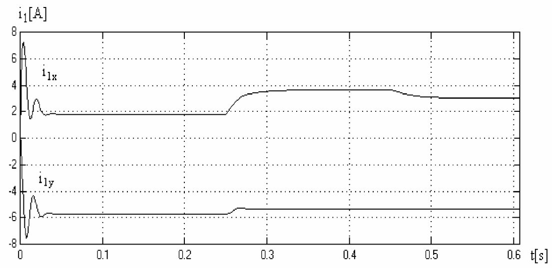

Fig. 1 Modelled waveforms of the stator current components for start-up, at t = 0,25s rated motor loading and at t = 0,45s motor derating

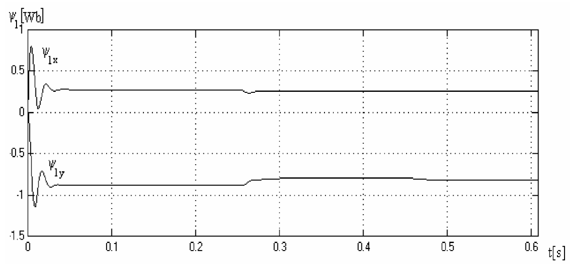

Fig. 2 Modelled waveforms of the stator magnetic linkage flux at start-up, at t= 0,25s rated loading of the motor and at t = 0,45s motor derating

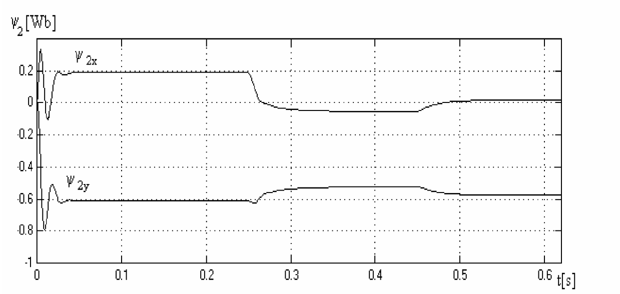

Fig. 3 Modelled waveforms of the rotor magnetic linkage flux at start-up, rated loading of the motor: at t = 0,25s and motor derating at t = 0,45s

The waveform of the modelled force generated by the motor for the case of motor unloaded start-up, at rated loading at t = 0,25s – 100% F_N, and at further changes in loading: at t = 0,45s – 75% F_N, at t = 0,65s –50% F_N and at t = 0,8s –25% F_N is shown in Fig. 4.

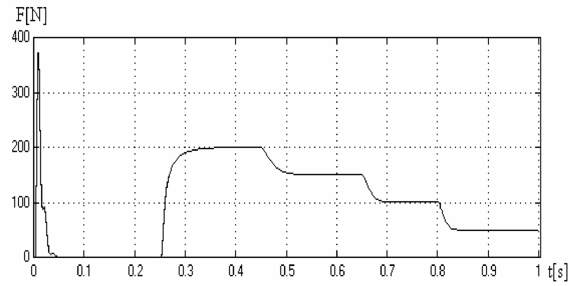

Fig. 4 Modelled waveforms of the magnetic force at start-up, rated loading of the motor: at t = 0,25s and further motor loading changes

4. MEASUREMENT AND COMPARISON

Measurements were provided on TLM-60 tubular linear induction motor with the above-mentioned parameters. Fig. 5 shows comparison of calculated and measured velocity *v(t)* of the tubular linear induction motor for starting mode.

Good coincidence between calculations and measurements shows that, notwithstanding the assumptions made, the system of equations in (16-21) or in (23-26) is useful for digital simulation and modelling of transients in electrical drives with tubular LIMs.

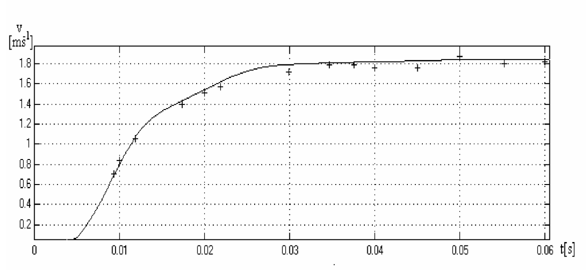

Fig. 5 Transient characteristics v(t) of the LIM for starting mode: - calculations, + measurement

5. CONCLUSION

In the paper, the mathematical model of a tubular linear induction motor is described by a set of differential equations and the state model, which are most conveniently solved using numerical methods. Good agreement between calculations and measurements proves that the mathematical model is useful for digital simulation and modelling of transients in electrical drives with tubular linear induction motors.

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BIOGRAPHY

Jaroslava Žilková (Ing., PhD.) received the MSc. degree at the Technical University of Košice, at the Department of Technical Cybernetics of the FEI. She finished her PhD. thesis, which dealt with estimation of induction motor variables based on artificial neural networks, in 2001 at the Department of Electrical Drives of the Faculty of Electrical Engineering and Informatics at the same university. Since 1991 she has been an assistant professor at the Department of Electrical Drives of the FEI TU Košice. Her field of interest is mathematical modelling and application of modern control methods to electrical drives, state estimation of AC using neural networks.

Jaroslav Timko (Prof., Ing., PhD.) has graduated in electrotechnics at Faculty of Electrical Engineering of ČVUT Praha. He received his CSc.(PhD.) degree at University of Žilina, in 1976. Since 1988 he has been professor at the Department of Electrical Drives of the Faculty of Electrical Engineering and Informatics at the Technical University of Košice. His research activities include the modern control strategies for industrial automation.