PWL VS. CONVENTIONAL MICROSTATISTIC DIGITAL FILTERS

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SUMMARY

In this paper we report a new approach to design of conventional microstatistic filters (CMF). This approach is based on the idea of describing the threshold decomposition operator by means of piecewise-linear functions. In particular we show that modification of the conventional mathematical model of the CMF yield a digital filters that can be considered to be a subset of the canonical piecewise-linear (CPWL) filters.

Keywords: *microstatistic filtering, piecewise-linear filtering, threshold decomposition*

1. INTRODUCTION

In this paper, new approach to design of CMF based on threshold decomposition operator expressed by means of piecewise-linear functions is shown. The aim of this approach is to compare the relationship between CMF and CPWL filters. In order to do this, the mathematical model of threshold decomposition, used as a segmentation operator in the CMF structure, is replaced by the piecewise-linear model – generally used for design of piecewise-linear (PWL) filters.

Two approaches of piecewise-linear signal modeling are examined here.

First approach – based on PWL filtering, approximate the nonlinear model by a locally linear functions defined on a partition of the domain into small subregions [4].

Another approach – based on microstatistic digital filtering, use threshold decomposition of the real valued discrete-time input signal to create a piecewise-linear signal model [1, 2, 3].

In section 2, a mathematical model of the canonical PWL filters is described. Next section introduces CMFs along with a multilevel threshold decomposition framework for the processing of real-valued signals. In section 4, a new representation of the threshold decomposition in the field of microstatistic digital filtering is presented. A comparison between the PWL and modified CMF approaches is given in section 5. Finally, in the conclusion some topics for the further research concerning microstatistic digital filtering are indicated.

2. PWL FILTERS APPROACH

Nonlinear system description based on piecewise-linear functions is a well-developed theory with a varied number of applications in signal processing [5]. It is tempting to draw upon linear signal processing theory, which is broad in application and mature in development, to devise non-optimal but practical solutions to nonlinear signal processing problems. Many such solutions rely on piecewise-linear models of a nonlinear signal.

PWL filters are nonlinear digital filters based on the approximation of nonlinear systems by piecewise-linear functions. They assume that the nonlinear operator can be represented as the union of multidimensional linear surfaces [4]. Their attractiveness comes from the fact that compared to the Volterra or neural network approaches, PWL filters are generally more economic in terms of the number of parameters required to achieve a good approximation. There are several techniques for building PWL filters.

One of them is based on regression trees [4]. The trees are typically binary. The effect of splitting the parent node into two branches is perceived as partitioning parent space into two subregions. Each of the subregions can be partitioned again in two new subdomains. The process continues resulting in a binary tree structure. The terminal nodes of the tree define a set of disjoint regions that correspond to partition of the original space. Ones the tree structure is defined, a linear filter is associated with each terminal node. Consequently, the whole tree structure plus the filters associated with the terminal nodes behave as a piecewise-linear filter. Tree structured filters have been successfully applied to problems in channel equalization and echo cancellation [4].

The combination of hinge planes is another method for building PWL approximations. A hinge function consists of two multivariate planes joined together. In this case, the nonlinear operator is built as the sum of several hinges, each of them having different orientation [4].

Another approach to PWL filtering and function approximation is referred to as Chua's canonical PWL functions. This theory solves the generalized problem of building piecewise-linear models based on any type of partition grids (not necessarily rectangular) [4]. Based on this approach, models of digital filters are based on finite expansions of linear systems combined through absolute value operators. This approach we use for the comparison with the modified CMF model.

Let { $x(n)$ } be a weakly stationary discrete-time real valued sequence and denote the N- long observation window of { $x(.)$ } as

$$\mathbf{x}(n) = [x(n), x(n-1), \cdots, x(n-N+1)]^T , \quad (1)$$

then Chua's canonical representation is given by

$$\hat{y}(n) = a + \mathbf{b}^T \mathbf{x}(n) + \sum_{i=1}^{S} c_i \left| \mathbf{d}_i^T \mathbf{x}(n) - \omega_i \right| , \quad (2)$$

where $\mathbf{b}$ and $\mathbf{d}_i$, $i = 1,2,\ldots,S$ are $N \times 1$ vectors and a , c_i and ω_i are scalar coefficients. All of these parameters, together with the number of terms S have to be estimated from the observed samples. Expression (2) represents the mathematical model of CPWL filter. This model will be used for comparison with the CMF model in next sections.

3. MICROSTATISTIC APPROACH

3.1. Conventional Threshold Decomposition

Threshold decomposition is a segmentation operator used to split a signal into a set of multilevel components – a set of decomposed signals [1, 2]. Each decomposed signal corresponds to an amplitude range of the input signal, where the limits of each range are determined by the threshold values. Each decomposed signal is fed into a digital filter and the outputs of the filters are summed to obtain the final filter output.

Let us assume a static $(L_D + 1)$-level threshold decomposer, whose input signal $x(n)$ given by expression (1) is decomposed into L_D output signals $x^{(k)}(n)$, $k = 1,\ldots,L_D$. In this way the threshold decomposer contains $L_D + 1$ threshold values (l_k for $k = 1,2,\ldots,L_D + 1$) confined as

$$\left(-\infty = l_{L_D+1}\right) < l_{L_D} < \cdots < l_k < \cdots < l_2 < \left(l_1 = \infty\right) . \quad (3)$$

The performance of the decomposer is given by

$$\mathbf{x}^{(k)}(n) = \left[x^{(k)}(n), x^{(k)}(n-1), \cdots, x^{(k)}(n-N+1)\right]^T \quad (4)$$

where $x^{(k)}(n)$ is the threshold decomposed sample at the k -th level at discrete time n .

Thus, the vector $\mathbf{x}^{(k)}(n)$ is uniquely determined from $\mathbf{x}(n)$ by

$$\mathbf{x}^{(k)}(n) = \left[D_k[x(n)], D_k[x(n-1)], \cdots, D_k[x(n-N+1)]\right]^T ,$$
for $k = 1,2,\ldots,L_D$ (5)

where $D_k[.]$ denotes the threshold decomposition operation of the k -th level. $D_k[x(n)]$ is for nonnegative values of input signal ($x(n) \geq 0$) given by

$$D_k[x(n)] = \begin{cases} 0 & \text{if} \quad x(n) \leq l_{k+1} \\ x(n) - l_{k+1} & \text{if} \quad l_{k+1} < x(n) \leq l_k \\ l_k - l_{k+1} & \text{if} \quad l_k < x(n) \end{cases} , \quad (6)$$

whereas $D_k[x(n)]$ is for negative values of input signal ($x(n) < 0$) given by

$$D_k[x(n)] = \begin{cases} 0 & \text{if} \quad l_k \leq x(n) \\ x(n) - l_k & \text{if} \quad l_{k+1} < x(n) \leq l_k \\ l_{k+1} - l_k & \text{if} \quad x(n) < l_{k+1} \end{cases} . \quad (7)$$

An example of threshold decomposition is shown in Fig. 1-3. Input signal of the decomposer $x(n)$ is illustrated in Fig. 1.

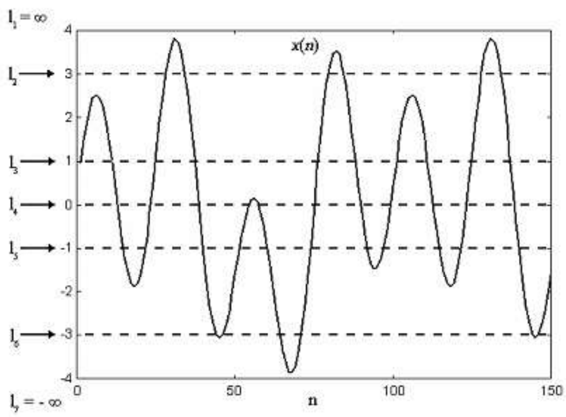

Fig. 1 Input signal of the threshold decomposer

Threshold values used for segmentation of $x(n)$ are listed in the Tab. 1.

l_1	l_2	l_3	l_4	l_5	l_6	l_7
∞	3	1	0	-1	-3	$-\infty$

Tab. 1 Threshold decomposition values

Threshold decomposed signals $x^{(k)}(n)$, $k = 1,\ldots,6$ for nonnegative half plane and negative half plane of $x(n)$ are depicted in Fig. 2 and Fig. 3, respectively.

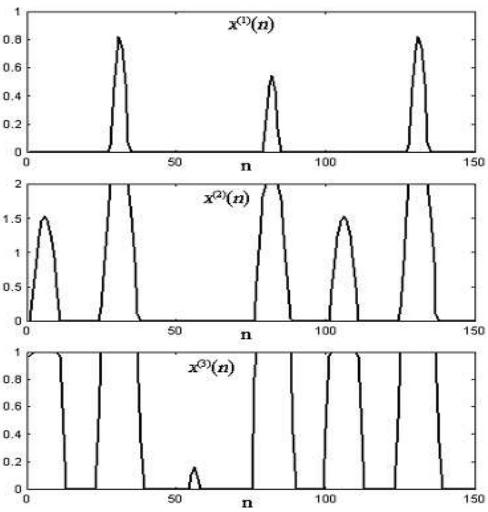

Fig. 2 Threshold decomposed signals $x^{(k)}(n)$, $k = 1,\ldots,3$ for nonnegative half plane

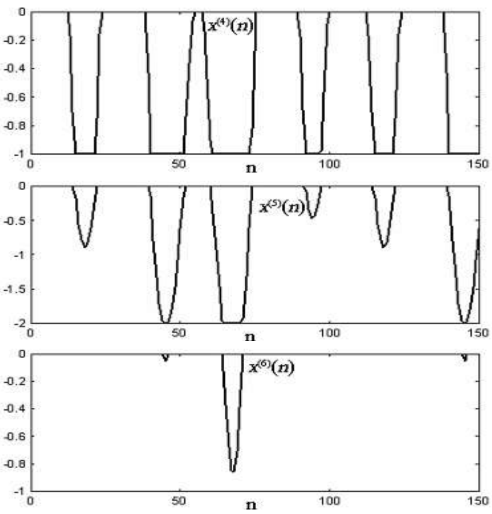

Fig. 3 Threshold decomposed signals $x^{(k)}(n)$, $k = 4,\ldots,6$ for negative half plane

3.2. Conventional Microstatistic Filters

CMFs [1, 2] are nonlinear estimators based on estimation of desired signal by using a linear combination of vector elements obtained by the threshold decomposition of signal $x(n)$. In conventional microstatistic filtering, each decomposed vector (5) is fed to an individual Wiener filter and the estimates of the desired result are summed. Block scheme of the CMF is given in the Fig. 4, where $x(n)$ and $\hat{y}(n)$ is the input and output signal of the CMF, respectively. It can be seen from Fig. 1 that the CMF consists of the threshold decomposer, L_D WFs and sumator. The k-th output signal of the threshold decomposer, given in (4), is fed into the k-th WF (WF$_k$). The output signal of the CMF is given by

$$\hat{y}(n) = h_0 + \sum_{k=1}^{L_D} \sum_{j=0}^{N-1} h_j^{(k)} x^{(k)}(n-j)\,, \tag{8}$$

where h_0 is constant term to be applied in CMF structure in order to obtain an unbiased CMF output and $h_j^{(k)}$ are the CMF weights containing all weights of WF$_k$ for $k = 1,\ldots,L_D$.

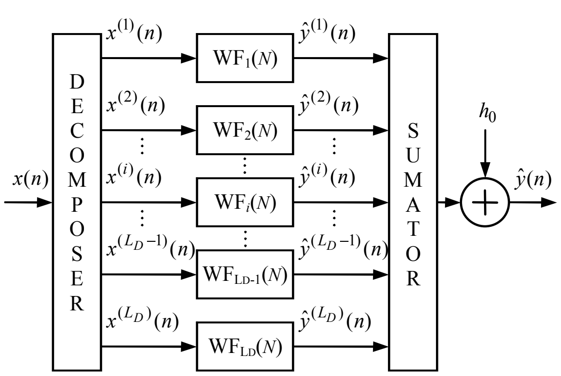

Fig. 4 Block diagram of CMF

4. NEW THRESHOLD DECOMPOSITION REPRESENTATION

In this section we define the mathematical model of the threshold decomposition given by (6) and (7) in the form of piecewise-linear functions. For the simplicity, let us modify the boundary thresholds defined by (3) as follows

$$l_1 = \max\{\, x(n) \,\}, \tag{9}$$
$$l_{L_D+1} = \min\{\, x(n) \,\}. \tag{10}$$

This modification does not violate the conditions in (3). "Empty space" only has been removed, in which the decomposed signals would be identically equal to zero ($x^{(k)}(n) = 0$).

Let us modify the conventional threshold decomposition operator $D_k[x(n)]$ given in (6) and (7) in the form of piecewise-linear functions. Let $\Psi_k[x(n)]$ be the new threshold decomposition operator. In order to describe CMF by the $\Psi_k[x(n)]$, following condition must be satisfied

$$D_k[x(n)] = \Psi_k[x(n)]. \tag{11}$$

Analysis of relations (6), (7), (9), (10) and (11) shows, that operator $\Psi_k[x(n)]$ can be given by

$$\Psi_k[x(n)] = s_k \frac{l_k - l_{k+1}}{2} - \frac{|x(n) - l_k| - |x(n) - l_{k+1}|}{2}\,, \tag{12}$$

where s_k is scalar coefficient that can be written as

$$s_k = \operatorname{sign}\left(l_k - l_{k+1}\right). \tag{13}$$

Fig. 5 and Fig. 6 show an example of the graphical representation of threshold operators $\Psi_k\left[x(n)\right]$ for nonnegative and negative half plane, respectively. The decomposition levels used in this example are listed in the Tab. 2.

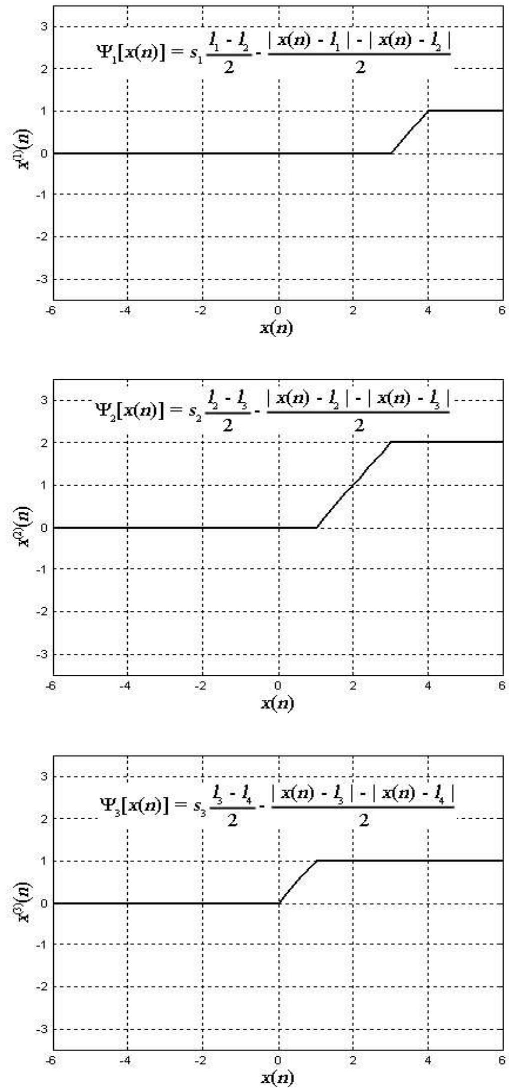

Fig. 5 Threshold operators $\Psi_k\left[x(n)\right],\ k = 1,\ldots,3$ for nonnegative half plane

l_1	l_2	l_3	l_4	l_5	l_6	l_7
4	3	1	0	-1	-3	-4

Tab. 2 Threshold decomposition values

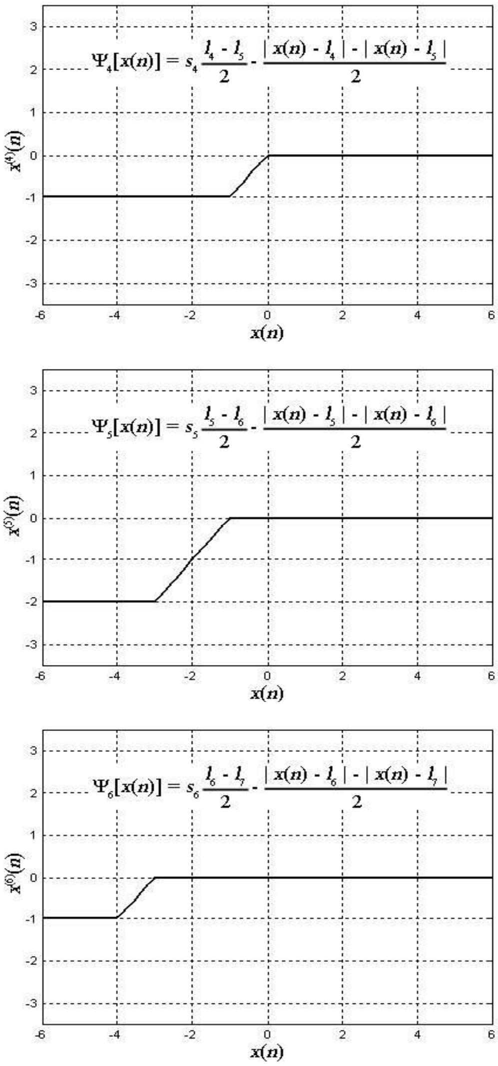

Fig. 6 Threshold operators $\Psi_k\left[x(n)\right],\ k = 4,\ldots,6$ for negative half plane

5. MODEL COMPARISON

In this section we define the mathematical model of the CMF based on piecewise-linear functions. This derivation will be compared with the Chua's canonical representation (2).

In order to define the output signal of the CMF following terms can be written as

$$\begin{aligned}
\mathrm{A} = {} & h_0 + s_1 \frac{l_1 - l_2}{2}\left(h_0^{(1)} + h_1^{(1)} + \cdots + h_{N-1}^{(1)}\right) + \\
& + s_2 \frac{l_2 - l_3}{2}\left(h_0^{(2)} + h_1^{(2)} + \cdots + h_{N-1}^{(2)}\right) + \cdots + \\
& + s_{L_D} \frac{l_{L_D} - l_{L_D+1}}{2}\left(h_0^{(L_D)} + h_1^{(L_D)} + \cdots + h_{N-1}^{(L_D)}\right),
\end{aligned} \tag{14}$$

$$\begin{aligned}
\mathrm{B}_1 = {} & \left(-\frac{h_0^{(1)}}{2}\right)\left|x(n) - l_1\right| - \frac{h_1^{(1)}}{2}\left|x(n-1) - l_1\right| - \\
& - \cdots - \frac{h_{N-1}^{(1)}}{2}\left|x(n - N + 1) - l_1\right|,
\end{aligned} \tag{15}$$

$$\mathrm{B}_2 = \frac{h_0^{(1)} - h_0^{(2)}}{2}\left|x(n) - l_2\right| + \frac{h_1^{(1)} - h_1^{(2)}}{2}\left|x(n-1) - l_2\right| + \\ + \cdots + \frac{h_{N-1}^{(1)} - h_{N-1}^{(2)}}{2}\left|x(n-N+1) - l_2\right|, \qquad (16)$$

$$\mathrm{B}_{L_D} = \frac{h_0^{(L_D-1)} - h_0^{(L_D)}}{2}\left|x(n) - l_{L_D}\right| + \frac{h_1^{(L_D-1)} - h_1^{(L_D)}}{2} \\ \left|x(n-1) - l_{L_D}\right| + \cdots + \frac{h_{N-1}^{(L_D-1)} - h_{N-1}^{(L_D)}}{2}\left|x(n-N+1) - l_{L_D}\right|, \qquad (17)$$

$$\mathrm{B}_{L_D+1} = \frac{h_0^{(L_D)}}{2}\left|x(n) - l_{L_D+1}\right| + \frac{h_1^{(L_D)}}{2}\left|x(n-1) - l_{L_D+1}\right| + \\ + \cdots + \frac{h_{N-1}^{(L_D)}}{2}\left|x(n-N+1) - l_{L_D+1}\right|. \qquad (18)$$

Let us define the $L_D \times 1$ a sign vector $\mathbf{s}$

$$\mathbf{s} = \left[s_1, s_2, \ldots, s_{L_D}\right]^T. \qquad (19)$$

Elements of the vector $\mathbf{s}$ have for the nonnegative half plane the value +1 and for the negative half plane the value –1.

Denote $\widetilde{\mathbf{L}}$ as $L_D \times 1$ difference vector by

$$\widetilde{\mathbf{L}} = \left[(l_1 - l_2),(l_2 - l_3),\ldots,\left(l_{L_D} - l_{L_D+1}\right)\right]^T. \qquad (20)$$

Next, define the weight vectors for decomposition level $k = 0$ and $k = L_D + 1$ in the form

$$\mathbf{H}^{(0)} = \left[h_0^{(0)}, h_1^{(0)}, h_2^{(0)}, \ldots, h_{N-1}^{(0)}\right]^T, \qquad (21)$$

for $x(n) > \max\{x(n)\}$ and

$$\mathbf{H}^{(L_D+1)} = \left[h_0^{(L_D+1)}, h_1^{(L_D+1)}, h_2^{(L_D+1)}, \ldots, h_{N-1}^{(L_D+1)}\right]^T, \qquad (22)$$

for $x(n) < \min\{x(n)\}$. It is noticed that

$$\mathbf{H}^{(0)} = \mathbf{H}^{(L_D+1)} = \mathbf{0}, \qquad (23)$$

where $\mathbf{0}$ is $N \times 1$ zero vector. Similarly, the weight vector can be given by

$$\widetilde{\mathbf{H}} = \left[\mathbf{H}^{(1)^T}\mathbf{H}^{(2)^T}\cdots\mathbf{H}^{(L_D)^T}\right]^T. \qquad (24)$$

Finally, the output signal of the CMF is given by

$$\hat{y}(n) = h_0 + \frac{1}{2}\widetilde{\mathbf{L}}^T \operatorname{diag}(\mathbf{s})\widetilde{\mathbf{H}}\mathbf{E} + \\ + \frac{1}{2}\sum_{i=1}^{L_D+1}\sum_{j=0}^{N-1}\left(h_j^{(i-1)} - h_j^{(i)}\right)\left|x(n-j-l_i)\right|, \qquad (25)$$

where $\mathbf{E}$ is $N \times 1$ identity vector. Equation (25) represents the mathematical model of CMF based on piecewise-linear functions.

6. CONCLUSION

In this paper, new approach to piecewise-linear signal modeling based on CMF with modified threshold decomposer have been presented. This framework has been compared with PWL system. This comparison has shown that CMFs can be considered to be a subset of the CPWL filters. This information can be helpful by the further research of microstatistic digital filtering, and leads to new areas of research in this field like channel equalization and echo cancellation [4].

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BIOGRAPHY

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