

## OPTIMAL DISTRIBUTION OF ACTIVE POWERS USING LINEAR PROGRAMMING WITH LOSSES COST MINIMIZATION

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### SUMMARY

*This paper deals with application of linear programming techniques to achieve a mixed economic distribution of the active powers (production cost of active power and cost of the losses of transmissions) in an electric power. Performance indexes related to energy production are non-linear. Newton method is used for linearising these cost functions around an operating point prior to their minimisation. In the first, the problem is solved by considering the fuel performance indexes only while maintaining transmission losses constant. In the second, we have considered transmission losses and production costs at the same time. We will also consider that the active losses are a function of the generated active powers and hence the coefficient of this function may be calculated with the Generalised Generation Factors Distribution (GGFD) method.*

**Keywords:** Function cost; active powers, load flow, optimal power flow, GGDF method's, losses of transmission line, linear programming.

## 1. INTRODUCTION

Nowadays, fuel cost necessary for the production of electrical energy becomes increasingly expensive. It is therefore necessary to reduce fuel costs in order to achieve the best production costs for the system.

An effective method which attempts to solve this problem consists of optimising the distribution of the active powers generated at the system control level. In this work, simplex method is applied on a network AEP 14 Bus System Test [6]. This network is composed of 14 nodes including 2 production nodes. Transmission losses were initially calculated using Gauss-Seidel method and maintained constant. Finally, minimisation of the total performance index of production is solved. After that, a relationship between transmission power losses and the generated active powers has been established and this time the task of minimisation which will be called mixed task of minimisation of the global performance index of energy production and the performance index of the losses of transmission is solved.

After that, we established a dependence of the losses of power of transmission according to the generated active powers and solved this time the task of minimization which we will call mixed task of minimization of the total performance index of energy production and the performance index of the losses of transmission.

## 2. MATHEMATICAL FORMULATIONS

The principal task which consists in modelling the electrical power network and minimising the total performance index of electrical energy production is formulated as

$$\text{Min} \left\{ \sum_{i=1}^{NG} F_i(P_{Gi}) \right\} \quad (1)$$

Under the following constraints:

$$\sum_{i=1}^{NG} P_{Gi} - P_{load} - P_L = 0 \quad (2)$$

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad i = 1 \div NG \quad (3)$$

$NG$ : numbers total nodes of production.

$P_{load}$ : the total consumption of the active power

$P_L$  : the total losses of active powers transmitted in the network

$P_{Gi}$  : active power generated at bus  $i$ .

$P_{Gi}^{\min}$  : minimal active power generated at bus  $i$ .

$P_{Gi}^{\max}$  : maximum generated active power at bus  $i$ .

$F_i(P_{Gi})$  : the cost function of fuel to bus  $i$

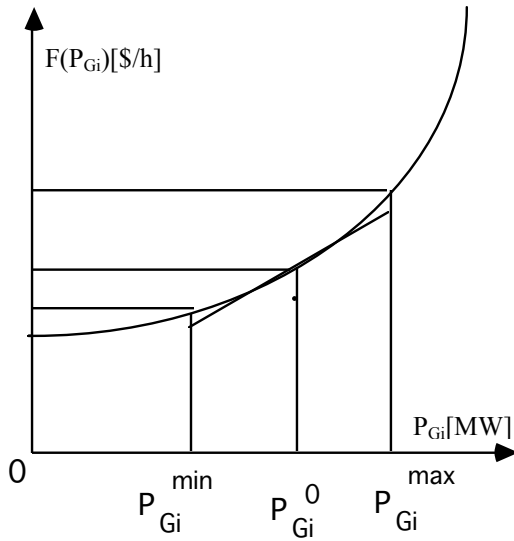
$P_{Gi}^0$  : point of the active power work at bus  $i$

As the performance index of production is form:

$$F(P_{Gi}) = a_i + b_i \cdot P_{Gi} + c_i \cdot P_{Gi}^2 \quad (4)$$

The simplex method will apply in order to use linear programming techniques, it will be necessary to replace the original function which is non-linear by a linear approximated function.

For this approximation, we will use Newton method [1] which consists in replacing part of the non-linear function by another linear function around a selected point called not work (Figure 1).



**Fig. 1** The function cost of fuel

That is valid of course only if one limits the interval in which the approximation is made. The new function to be minimised is obtained in the following way.

$$F_i = \frac{\partial F_i(P_{Gi}^0)}{\partial P_{Gi}} P_{Gi} + A_i \quad (5)$$

Where:

$$K_i = \frac{\partial F_i(P_{Gi}^0)}{\partial P_{Gi}} \quad (6)$$

And  $A_i$  is a constant.

As we must associate the cost of the losses with the production cost of the active powers [2], [5] which is being assumed as a linear function, the final minimisation task becomes:

$$\text{Min} \left\{ \sum_{i=1}^{NG} K_i \cdot P_{Gi} + h \cdot P_L \right\} \quad (7)$$

Under the following constraints:

$$\sum_{i=1}^{NG} P_{Gi} - P_{load} - P_L = 0 \quad (8)$$

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max} \quad (9)$$

Where  $h$  is a constant coefficient.

### 3. PRESENTATION OF METHOD G.G.D.F. [4]

The total active losses in a network are given by the following relation:

$$P_L = \sum_{i=1}^N P_i = \sum_{i=1}^N \sum_{j=1}^N P_{ij} \quad (10)$$

Where:

$N$  is the total bus in power system

$P_i$  is the active power injected at bus  $i$ .

The power transmitted in the line  $(i,j)$  is given by

$$P_L = \sum_{k=1}^{NG} D_{ij,k} * P_{GK} \quad (11)$$

With  $i,j = 1 \dots N$  and  $k \dots R$

$R$  is the number of slack bus

$D_{ij,k}$  is the GGDF coefficient of the line  $(i,j)$  compared to the generator  $k$ .

$$D_{ij,R} = \frac{P_{ij} - \sum_{k=1}^{NG} P_i * P_{GK}}{\sum_{g=1}^{NG} P_{Gg}} \quad (12)$$

And

$$D_{ij,R} = \frac{X_{ik} - X_{jk}}{x_{ij}} \text{ with } R \neq k \quad (13)$$

Where,

$X_{ik}, X_{jk}$  : Reactance of the nodal matrix of impedance.

$X_{ij}$  : Reactance of the line  $ij$ .

$P_{ij}$  and  $P_{GK}$  are calculated by Gauss-Seidel method by replacing  $P_{ij}$  by its value given by the relation of the losses  $P_L$  which leads to the expression of the losses according to the generated powers:

The linear formula of losses according to the generated powers is thus given by the following relation

$$P_L = \sum_{k=1}^{NG} \sum_{i=1}^N \sum_{j=1}^N D_{ij,k} * P_{Gk} \quad (14)$$

Finally we obtain a linear relation of the losses according to the generated powers.

$$P_L = \sum_{k=1}^{NG} K_k * P_{Gk} \quad (15)$$

#### 4. EXPERIMENTATION

As underlined earlier, the application is performed on the IEEE14 system has been selected to assess the correctness of the proposed model and its implementation [5]. Whose diagram is illustrated by Figure 2 including two nodes of production and 13 nodes of consumption:

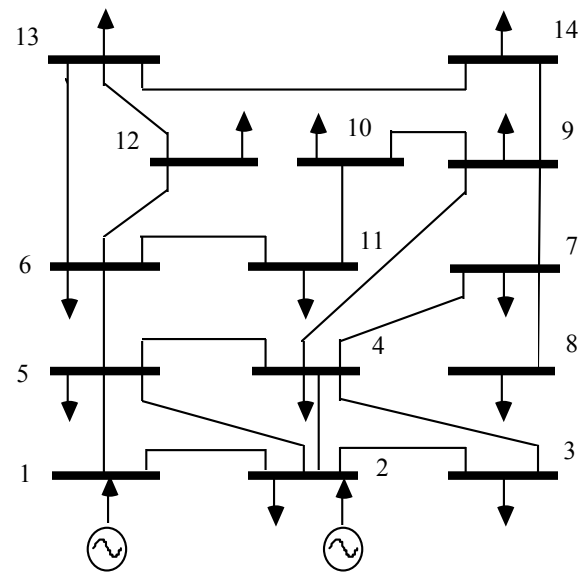
Impedance and line charging		
p-q	$Z_{km}$ impedance	$y_{sh_{km}}$
1-2	0.01938+j0.05917	j0.0264
1-5	0.05403+j0.22304	j0.0246
2-3	0.04699+j0.19797	j0.0219
2-4	0.05811+j0.17632	j0.0187
2-5	0.05695+j0.17388	j0.0170
3-4	0.06701+j0.17103	j0.0170
4-5	0.01335+j0.04211	j0.0006
4-7	0.00000+j0.02091	j0.0000
4-9	0.00000+j0.55618	j0.0000
5-6	0.00000+j0.25202	j0.0000
6-11	0.09498+j0.19890	j0.0000
6-12	0.12291+j0.25581	j0.0000
6-13	0.06615+j0.13027	j0.0000
7-8	0.00000+j0.17615	j0.0000
7-9	0.00000+j0.11001	j0.0000
9-10	0.03181+j0.08450	j0.0000
9-14	0.12711+j0.27038	j0.0000
10-11	0.08205+j0.19207	j0.0000
12-13	0.22092+j0.19988	j0.0000
13-14	0.17093+j0.34802	j0.0000

**Tab. 1** Impedance and line charging for IEEE14 system

P	type of bus	P (p.u)	Q (p.u)
1	slack	0	0
2	production	0.6825	0.2489
3	consumption	-0.950	-0.1079
4	"	-0.490	-0.037
5	"	-0.078	-0.020

6	"	-0.1199	-0.1535
7	"	-0.020	-0.010
8	"	-0.1099	-0.2162
9	"	-0.310	-0.180
10	"	-0.110	-0.070
11	"	-0.045	-0.020
12	"	-0.070	-0.018
13	"	-0.140	-0.060
14	"	-0.160	-0.060

**Tab. 2** Table of specified values



**Fig. 2** The IEEE14 system

The performance indexes of the two nodes of production are:

$$F_1(P_{G1}) = 100 + 1.5 \cdot P_{G1} + 0.006 \cdot P_{G1}^2 \quad (16)$$

$$F_2(P_{G2}) = 130 + 2.1 \cdot P_{G2} + 0.009 \cdot P_{G2}^2 \quad (17)$$

With the following constraints:

$$135 \leq P_{G1} \leq 195 \text{ (MW)} \quad (18)$$

$$70 \leq P_{G2} \leq 145 \text{ (MW)} \quad (19)$$

$$P_{load} = 251.8 \text{ MW} \quad (20)$$

With:

$$U_b = 220 \text{ KV} \quad (21)$$

$$S_b = 100 \text{ MVA} \quad (22)$$

Where  $U_b$ : is the basic value of bus voltage  
 $S_b$ : is the basic value of Power

#### 4.1. First alternative

The calculation of the loads flow by Gauss-Seidel method gives the following value:

$$P_{G1} = 20.6 \text{ MW} \quad (23)$$

There are the total losses of the network. The equation of assessment which is an equality constraint becomes:

$$P_{G1} + P_{G2} = 272.4 \text{ MW} \quad (24)$$

The following points of work are:

$$P_{G1}^0 = 180 \text{ MW} \text{ and } P_{G2}^0 = 90 \text{ MW} \quad (25)$$

The task of minimization becomes:

$$\text{Min } \{3.66P_{G1} + 3.72P_{G2}\} \quad (26)$$

Under the following constraints:

$$P_{G1} + P_{G2} = 272.4 \text{ MW} \quad (27)$$

$$135 \leq P_{G1} \leq 195 \text{ (MW)}$$

$$70 \leq P_{G2} \leq 145 \text{ (MW)} \quad (28)$$

#### 4.2. Second Alternative

For the determination of h; we do several tests (see the table following)

$$\text{With: } h = \frac{\Delta F^{\text{opt}}}{P_L} \quad (29)$$

Where  $\Delta F^{\text{opt}}$  represent the difference between the two optimal values of powers with and without losses

It is noticed that the value of the coefficient h depend of the point of work selected. For our study we chose the average value of h, calculated according to the table precedent (Table 4).

$$h = 3.75 \quad (\$/\text{MW}^2\text{h})$$

$P_{G1}^0$ (MW)	$P_{G2}^0$ (MW)	$K_1$	$K_2$	Without losses			With losses			$\Delta F^{\text{opt}}$ (\$/MWh)	h (\$/MW <sup>2</sup> h)
				$P_{G1}^{\text{opt}}$ (MW)	$P_{G2}^{\text{opt}}$ (MW)	$F^{\text{opt}}$ (\$/MWh)	$P_{G1}^{\text{opt}}$ (MW)	$P_{G2}^{\text{opt}}$ (MW)	$F^{\text{opt}}$ (\$/MWh)		
190	85	3.78	3.63	135	125.28	965.06	135.88	145	1039.97	74.91	3.63
180	90	3.68	3.72	190.280	70	960.63	195	85.88	1037.07	76.44	3.70
170	90	3.54	3.72	190.28	70	933.99	195	85.88	1009.7	75.71	3.67
150	100	3.30	3.90	190.88	70	900.92	195	85.88	978.43	77.51	3.76
145	115	3.24	4.17	190.28	70	908.40	195	85.88	989.91	81.51	3.95

**Tab. 4** Table of comparison

In this case, we express the losses as a function of the generated powers. The results obtained by G.G.DF method lead to the values of the coefficients of this function:

$$P_L = 0.0675.P_{G1} + 0.0249.P_{G2} \quad (30)$$

By taking the same values of the points of work as the first alternative, the task of minimization will become then (31).

$$\text{Min } \{3.91313P_{G1} + 3.813385P_{G2}\} \quad (31)$$

Under the following constraints:

$$135 \leq P_{G1} \leq 195 \text{ (MW)} \quad (32)$$

$$135 \leq P_{G2} \leq 195 \text{ (MW)} \quad (33)$$

$$P_{G1} + P_{G2} = 274.4 + P_L$$

$$P_L = 0.0675P_{G1} + 0.0249P_{G2} \quad (34)$$

$$(1 - 0.0675)P_{G1} + (1 - 0.0249)P_{G2} = 272.4$$

$$0.9325P_{G1} + 0.9751P_{G2} = 272.4 \quad \text{MW}$$

#### 4.3. Results and comparison

By using the linear programming (method of the simplex), we obtained the following results.

Designation	1 <sup>st</sup> alternative	2 <sup>nd</sup> alternative
$P_{G1}^{\text{opt}}$ (MW)	195	135
$P_{G2}^{\text{opt}}$ (MW)	77.4	129.1278
$F_{\text{min}}$ (\$/MWh)	1001.628	996.82
Computing time (s)	1	1
No. of iterations	5	4

**Tab. 3** Results

According to the results obtained, it can be said that it is interesting to regard the losses as function of the generated powers. The mixed task of minimisation gave better results that the first alternative to the level of the production cost of the active power.

## 5. CONCLUSION

With through resulted found, we can say that it is interested at the losses transmitted as being a function of the powers generated, the mixed spot of minimization give better results that the first alternative which consists in making only one optimization on the level of the production cost of the active power. These results which are very significant can be used by Sonelgaz for the economic control system for the national level

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## BIOGRAPHIES

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