ELASTOMAGNETIC SENSOR FIELD DETERMINATION USING MATLAB

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SUMMARY

In the presented paper we deal with the determination of magnetic field in the elastomagnetic sensor core. The elastomagnetic sensor core consists of the ferromagnetic plates which thickness is negligible to the rest of the core dimensions so it is sufficient to solve the magnetic field in one core plate. The dimension of the plate is designed in such a way that the plate can be divided into the squares a number of which is equal to a number of the holes in each plate. Each of the square is called as an integrative element of the sensor and the magnetic field in such one is solved.

The magnetic field in this integrative element is computed by using Partial Differential Equation Toolbox of MATLAB that provides effectual tool for the solution of partial differential equations in two space dimensions and time. The use of this toolbox requires only a basic level of the partial differential equation knowledge what is considered to be the most important advantage of this toolbox. The magnetic field in the integrative sensor element for the plane case can be expressed by the elliptic partial differential equation for the magnetic vector potential. The partial differential equation coefficients are magnetic permeability and the current density for the domain in which the magnetic vector potential field is solved. The domain consists of three subdomains so the coefficients in each of these regions have to be determined. For the subdomain that represents ferromagnet the functionality between the magnetic permeability and the magnetic flux density is needed because ferromagnet permeability is dependent on the field strength. The boundary conditions specifying for the outer boundaries and the interior ones is the necessary part of the PDE problem formulation too.

The toolbox provides execution the most of hard work such as the mesh generation, the partial differential equation transformation to the discrete form by the finite element method and the approximation to the solution itself. The results can be visualized in several ways. In our case the equipotential lines of magnetic vector potential in the integrative sensor element are selected.

Keywords: *magnetic field modelling, elliptic partial differential equation, finite element method, elastomagnetic sensor, ferromagnetic material.*

1. INTRODUCTION

MATLAB is an interactive numerical computation program. It contains powerful built-in routines that enable a very wide variety of computations. The graphics commands can be easily used too so the visualization of obtained results is immediately available. Very important part of MATLAB are the toolboxes that make the solution of specific applications possible for their users. One of them is the Partial Differential Equation (PDE) Toolbox that provides an effective environment for the solution of partial differential equations in two space dimensions and time. The solution is based on the discretization of this equation by the Finite Element Method (FEM) and its numerical calculation.

This toolbox was used for the magnetic field determination of a pressure force elastomagnetic sensor.

2. ELASTOMAGNETIC SENSOR

2.1. Elastomagnetic effect

An elastomagnetic sensor operates on the Villari effect principle of which is based on the proportionality between change of the ferromagnetic material and acting mechanical stress. The permeability increment $\nabla\mu$ proportional to the mechanical stress σ can be described by relation

$$\nabla\mu = \frac{2\lambda_{ms}}{B_s^2}\mu^2\sigma ,$$

where λ_{ms} is a coefficient of magnetostriction for $B = B_s$, B_s is the saturation flux density, μ is the magnetic permeability. The permeability increment causes the voltage change in the output of the elastomagnetic sensor.

2.2. Description of the elastomagnetic sensor

The elastomagnetic sensor for pressure force measurement consists of two basic parts: ferromagnetic core with couple of holes system and excitation coils system as it is illustrated in Fig. 1. A simplifying representation of the sensor core is in Fig. 2. The core consists of n = 50 ferromagnetic plates which thickness h = 0.5 mm is negligible in comparison with their other dimensions. Each of the plates has m holes with radius a = 0.001 m. arranged equidistantly on the cross axis of the plate and the distance of their centers is $2b$ = 0.012 m. All dimensions of the plates are designed in such a way that each of them can be divided into m = 4 squares we called integrative sensor element. In cartesian coordinates the spatial arrangement of the core magnetic field will depend only on two coordinates and doesn't depend on this coordinate which axis is parallel to the current wire wound between the couples of m = 4 holes and created excitation coil system. So the solution of the magnetic field in the sensor core is reduced to the solution of the magnetic field in one plate. Further, the dimensions of the plates are designed in such a way that each of them can be divided into m squares, called as the integrative sensor element (Fig. 3). If the influence of the leakage magnetic flux at the plate edges is neglected the resultant magnetic field in the plate is a m – multiple of the magnetic field in one square.

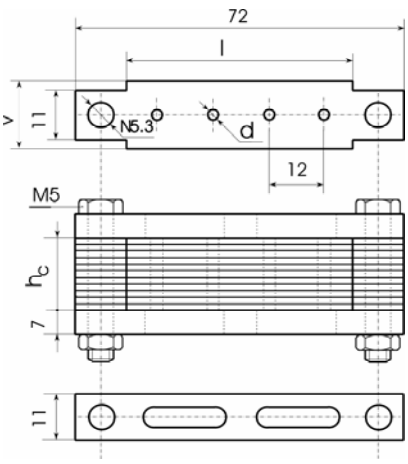

Fig. 1 The pressure force elastomagnetic sensor.

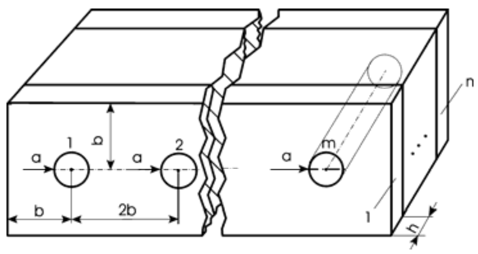

Fig. 2 A pressure force elastomagnetic sensor core in simplified form.

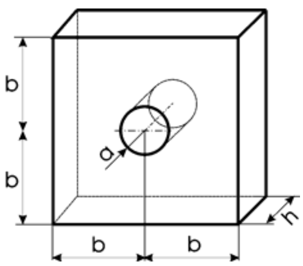

Fig. 3 An integrative element of pressure force elastomagnetic sensor.

3. EQUATION FOR MAGNETIC VECTOR POTENTIAL DETERMINATION

Maxwell's equations for steady case (the time rate of change is slow) are:

$$rot\,\mathbf{H} = \mathbf{J} \tag{1}$$

$$div\,\mathbf{B} = 0 \tag{2}$$

further, for a nonlinear magnetic environment is

$$div\ \mathbf{H} = -\frac{\mathbf{H}\ grad\ \mu(\mathrm{r})}{\mu(\mathrm{r})} \tag{3}$$

and material equation is

$$\mathbf{B} = \mu\,\mathbf{H} \tag{4}$$

where $\mathbf{H}$ is the magnetic field intensity, $\mathbf{J}$ is the current density, $\mathbf{B}$ is the magnetic flux density, and μ is the material's magnetic permeability.

Since $div\,\mathbf{B} = 0$, there exists a magnetic vector potential $\mathbf{A}$ such that

$$\mathbf{B} = rot\,\mathbf{A} \tag{5}$$

and

$$rot\,(\frac{1}{\mu} rot\,\mathbf{A}) = \mathbf{J} \tag{6}$$

For the plane case we assume that the current flows are parallel to the z-axis, it means that

$$\mathbf{J} = \mathbf{k}\,J$$

so only the z component of $\mathbf{A}$ is present,

$$\mathbf{A} = \mathbf{k}\,A$$

and the equation (6) we can simplify to the scalar elliptic PDE

$$-div\,(\frac{1}{\mu} div\,A) = J \tag{7}$$

where $J = J(x, y)$.

For the plane case, the magnetic flux density $\mathbf{B}$ can be computed as

$$\mathbf{B} = \mathbf{i}\frac{\delta A}{\delta y} + \mathbf{j}(-\frac{\delta A}{\delta x}) \tag{8}$$

and the magnetic field intensity $\mathbf{H}$ is

$$\mathbf{H} = \frac{1}{\mu}\mathbf{B} \tag{9}$$

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4. FORMULATION PDE PROBLEM FOR INTEGRATIVE SENSOR ELEMENT

The PDE problem formulation for the integrative sensor element requires determining the domain in which the magnetic field will be solved, to write the boundary conditions and to formulate the PDE specification.

4.1. The domain definition

The domain in which the magnetic field will be solved is the integrative sensor element, so it consists of three regions (Fig.4):

- the current wire (subdomain 2),
- the ferromagnetic sensor core (subdomain 1),
- the air gap between the wire and ferromagnetic core (subdomain 3),

so that 2-D geometry of the integrative sensor element is created.

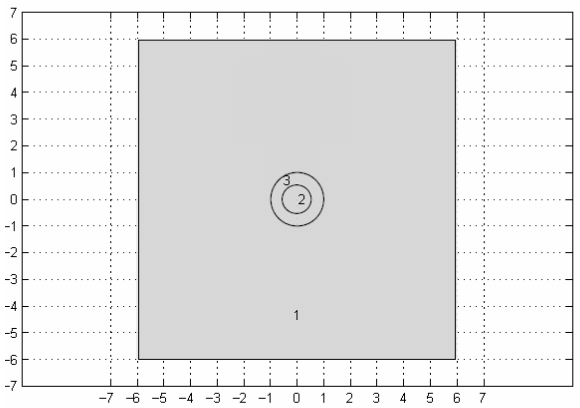

Fig. 4 The domain in which elliptic PDE is solved.

4.2. The boudary conditions

The Dirichlet boundary condition specifies the magnetic vector potential value on the exterior boundary. For our case the Dirichlet boundary condition is equal zero on the exterior boundary. The interface conditions between regions of different material properties have to satisfy the Neumann boundary condition, i.e. the continuity of $\frac{1}{\mu}\frac{\partial A}{\partial n}$, which does not require special treatment since the variational formulation of the PDE problem is used.

4.3. PDE specification

For determining the magnetic field in the integrative sensor element the scalar elliptic PDE (7) will be used, so the magnetic permeability μ and the current density J must be defined in each of three regions from which the domain consists of.

The magnetic permeability in the air and in the copper wire is equal μ_0 and in the ferromagnetic sensor core it depends on the field strength B, so the B – H curve must be measured for computation of ferromagnetic material permeability. For the obtained set of data points (μ, B) the curve fitting by fourth order polynomial

$$\mu(B) = -0.032834\,B^4 + +0.058663\,B^3 - \\ -0.040264\,B^2 + 0.012777\,B + 0.0011342$$

is made (Fig. 5).

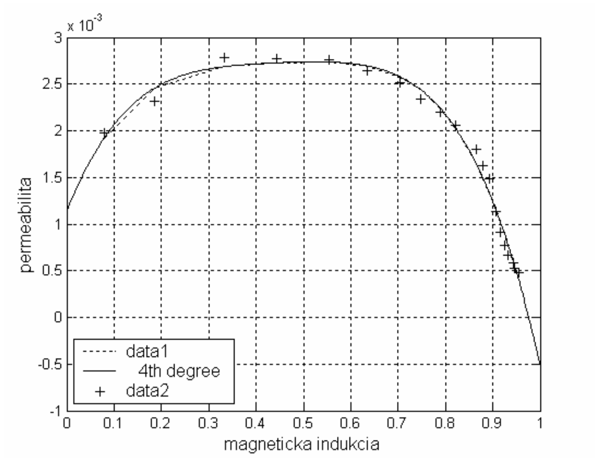

Fig. 5 The curve fitting $\mu = \mu(\mathrm{B})$.

The current density is equal 0 everywhere except the current wire, where it is 13.518 A/mm^2.

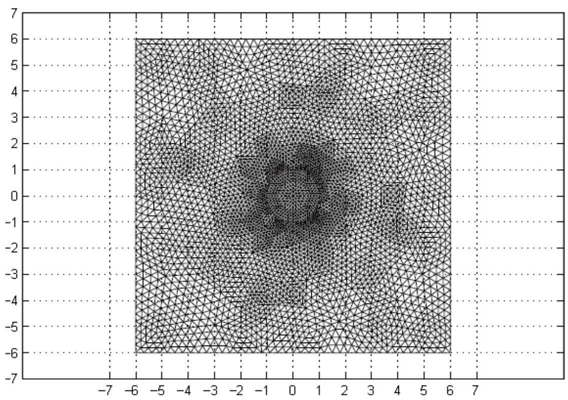

Fig. 6 A triangular mesh for the domain in which the magnetic field is solved.

5. NUMERICAL SOLUTION PDE PROBLEM OF AN INTEGRATIVE SENSOR ELEMENT

After creating 2 – D geometry for our PDE problem, specification the boundary conditions, specification of PDE type and determination of the PDE coefficients for each subdomain independently, the automated mesh generator is initialized to generate and plot the triangular mesh.

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For our domain we decided that a number of triangles in the mesh would be 11712 (Fig. 6).

Then the parameters μ and *J* for solving PDE are selected. Because μ is nonlinear in the sensor core, our PDE problem is a nonlinear one and the nonlinear solver has to be invoked. The tolerance parameter 1.0 E-6 is adjusted for this solver. After solving our PDE the equipotential lines of the solution – the magnetic vector potential – are automatically plotted using a contour plot (Fig. 7).

For our PDE problem we decided to draw the magnetic flux density field too. This field is visualized by using arrows in the Fig. 8. The plot clearly shows, as expected, that the magnetic flux density lines are parallel to the equipotential lines of the magnetic vector potential.

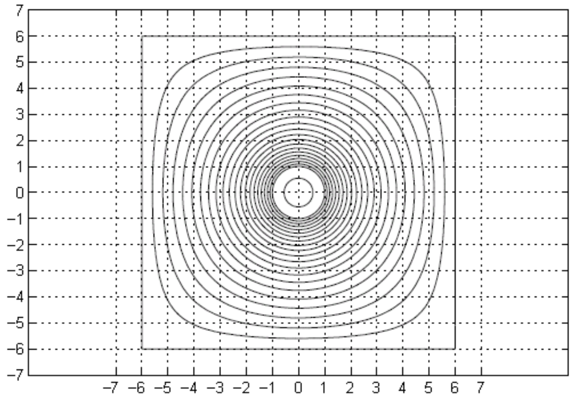

Fig. 7 Equipotential lines of magnetic vector potential in the integrative sensor element

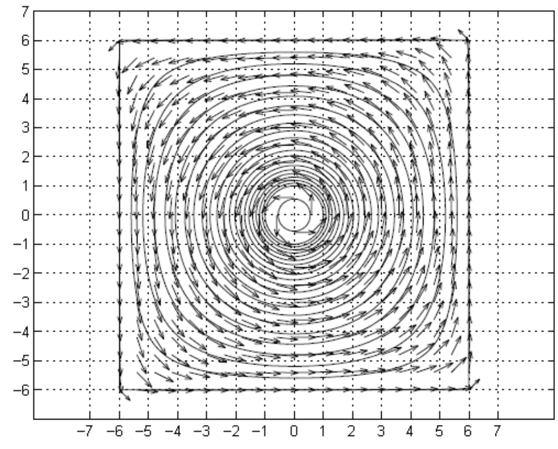

Fig. 8 Magnetic flux density field in the integrative sensor element

6. CONCLUSION

The magnetic field in the integrative element of the elastomagnetic sensor was solved in this paper without the force application by now. The magnetic vector potential field of this element is represented by the elliptic partial differential equation which analytical solution is complicated and challenging task in generally and requires above standard knowledge of this part of mathematics. The numerical solution of the PDE using PDE Toolbox requires the minimal knowledge about the partial differential equations and their solution. It requires only a correct formulation of a PDE problem – to draw and to characterize the domain, to write the boundary conditions and specify the PDE.

The obtained solution for the magnetic field in our elastomagnetic sensor integrative element is in full accord with our theoretical assumptions and experimental results.

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BIOGRAPHIES

Iveta Tomčíková was born in 1959. In 1983 she graduated (MSc.) at the Department of Technical Cybernetics of the Faculty of Electrical Engineering and Informatics at Technical University in Košice. She defended her PhD. in the field of Measurement Technology from the Slovak Technical University, Bratislava, in 1995. Since 1983 she has been with the Technical university, Košice, Faculty of Electrical Engineering and Informatics, Department of Theoretical Electrical Engineering and Measurement. Her field of interest includes circuit theory, theory of electromagnetism, mathematical models and her scientific research is focusing on the elastomagnetic sensors.

Darina Špaldonová (doc, RNDr, CSc) was born in Košice in 1954. In 1976 she graduated at P. J. Šafárik University in Košice, in 1983 received the RNDr (MSc) degree at this university and in 1994 CSc (PhD) degree in theory of electrical engineering at Slovak University of Technology, Bratislava. In 1998 she was appointed Associate Professor of theory of electrical engineering at the Department of Theoretical Electrical Engineering and Electrical Measurement of the Technical university of Košice. Her current interests include circuit theory and theory of electromagnetic field.