MUNICIPAL CREDITWORTHINESS MODELLING BY NEURAL NETWORKS

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ABSTRACT

The paper presents the design of municipal creditworthiness parameters. Further, the design of model for municipal creditworthiness classification is presented. The realized data pre-processing makes the suitable economic interpretation of results possible. Municipalities are assigned to clusters by unsupervised methods. The combination of Kohonen's self-organizing feature maps and K-means algorithm is a suitable method for municipal creditworthiness modelling. The number of classes in this model is determined by indexes evaluating the quality of clustering. The model is composed of Kohonen's self-organizing feature maps and fuzzy logic neural networks, where the output of Kohonen's self-organizing feature maps represents the input of fuzzy logic neural networks. The suitability of the designed model is compared with other fuzzy logic based classifiers. The highest classification accuracy is achieved by the fuzzy logic neural network. The fuzzy logic neural network's model is the best in terms of generalization ability due to this fact. By the designed model high classification accuracy, low average classification error and suitable interpretation of results are achieved..

Keywords: *municipal creditworthiness, Kohonen's self-organizing feature maps, fuzzy logic neural networks*

1. INTRODUCTION

Municipal creditworthiness [8] is the ability of a municipality to meet its short-term and long-term financial obligations. It is determined by factors (parameters) relevant to the assessed object. High municipal creditworthiness shows a low credit risk, while the low one shows a high credit risk. Municipal creditworthiness evaluation is currently being realized by methods combining mathematical-statistical methods and expert opinion [8]. Their output is represented either by a score evaluating the municipal creditworthiness (scoring systems) or by an assignment of the i-th object $o_i \in O$, $O=\{o_1,o_2, \dots ,o_i, \dots ,o_n\}$ to the j-th class $\omega_{i,j} \in \Omega$, $\Omega=\{\omega_{1,j},\omega_{2,j}, \dots ,\omega_{i,j}, \dots ,\omega_{n,j}\}$ according to their creditworthiness (rating, unsupervised methods). Rating is an independent expert evaluation based on complex analysis of all known risk parameters of municipal creditworthiness, however, it is considered to be rather subjective.

Municipalities are classified into classes $\omega_{i,j} \in \Omega$ by rating-based models. The classes are assigned to the municipalities by rating agencies. Therefore, the methods capable of processing and learning the expert knowledge, enabling their user to generalize and properly interpret, have proved to be most suitable. For example, fuzzy inference systems [7] and neural networks [3] are suitable for municipal creditworthiness evaluation.

Neural networks are appropriate for municipal creditworthiness modelling due to their ability to learn, generalize and model non-linear relations. Nevertheless, the computational speed and robustness are retained. Municipal creditworthiness evaluation is considered to be a problem of classification, that is, it can be realized by various models of neural networks. They differ in their topology and learning process. Advantages of both methods can be gained by neuro-fuzzy systems [6].

The paper presents the design of municipal creditworthiness parameters. Only those parameters were selected which show low correlation dependences. Therefore, data matrix **P** is designed where vectors $\mathbf{p}_i$ characterize municipalities $o_i \in O$. Further, the paper presents the basic concepts of the Kohonen's self-organizing feature maps (SOFMs) and fuzzy logic neural networks (FLNNs). The contribution of the paper lies in the model design for municipal creditworthiness evaluation. The model realizes the advantages of both the unsupervised methods (combination of the SOFM and K-means algorithm) and FLNN. The final part of the paper includes the analysis of the results.

2. MUNICIPAL CREDITWORTHINESS PARAMETERS DESIGN

In [2] common categories of parameters there are mentioned namely economic, debt, financial and administrative categories. The economic, debt and financial parameters are pivotal. Economic parameters affect long-term credit risk. The municipalities with more diversified economy and more favourable social and economic conditions are better prepared for the economic recession. Debt parameters include the size and structure of the debt. Financial parameters inform about the budget implementation. Their values are extracted from the municipality budget. The design of parameters, based on previous correlation analysis [2] and recommendations of notable experts, can be realized as presented in Table 1.

Based on the presented facts, the following data matrix **P** can be designed

$$\mathbf{P} = \begin{array}{c|ccccc:c} & x_1 & \dots & x_k & \dots & x_m & \omega \\ \hline o_1 & x_{1,1} & \dots & x_{1,k} & \dots & x_{1,m} & \omega_{1,j} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ o_i & x_{i,1} & \dots & x_{i,k} & \dots & x_{i,m} & \omega_{i,j} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ o_n & x_{n,1} & \dots & x_{n,k} & \dots & x_{n,m} & \omega_{n,j} \end{array},$$

where $o_i \in O$, $O=\{o_1,o_2, \dots ,o_i, \dots ,o_n\}$ are objects (municipalities), x_k is the k-th parameter, $x_{i,k}$ is the value of the parameter x_k for the i-th object $o_i \in O$, $\omega_{i,j}$ is the j-th class assigned to the i-th object $o_i \in O$, $\mathbf{p}_i = (x_{i,1}, x_{i,2}, \dots, x_{i,k}, \dots, x_{i,m})$ is the i-th pattern, $\mathbf{x} = (x_1, x_2, \dots, x_m)$ is the parameters vector.

Table 1 Municipal creditworthiness parameters

	Parameters
Economic	$x_1 = PO_r$, PO_r is population in the r-th year.
	$x_2 = PO_r/PO_{r-s}$, PO_{r-s} is population in the year r-s, and s is the selected time period.
	$x_3 = U$, U is the unemployment rate in a municipality.
	$x_4 = \sum_{i=1}^{k} (PZO_i/PZ)^2$, PZO_i is the employed population of the municipality in the i-th economic sector, i=1,2, … ,k, PZ is the total number of employed inhabitants, k is the number of the economic sector.
Debt	$x_5 = DS/OP$, $x_5 \in <0,1>$, DS is debt service, OP are periodical revenues.
	$x_6 = CD/PO$, CD is a total debt.
	$x_7 = KD/CD$, $x_7 \in <0,1>$, KD is short-term debt.
Financial	$x_8 = OP/BV$, $x_8 \in R^+$, BV are current expenditures.
	$x_9 = VP/CP$, $x_9 \in <0,1>$, VP are own revenues, CP are total revenues.
	$x_{10} = KV/CV$, $x_{10} \in <0,1>$, KV are capital expenditures, CV are total expenditures.
	$x_{11} = IP/CP$, $x_{11} \in <0,1>$, IP are capital revenues.
	$x_{12} = LM/PO$, [Czech Crowns], LM is the size of the municipal liquid assets.

3. DESIGN OF MODEL BASED ON NEURO-FUZZY SYSTEMS

Municipal creditworthiness modelling is realized by a design of the model. Data pre-processing makes the suitable economic interpretation of results possible. Municipalities are assigned to clusters by unsupervised methods. The output of SOFMs are used as the inputs of FLNN. Classification of municipalities into classes is realized this way.

Municipal creditworthiness modelling represents a classification problem. It is generally possible to define it this way:

Let $F(\mathbf{x})$ be a function defined on a set A, which assigns picture $\hat{x}$ (the value of the function from a set B) to each element $\mathbf{x} \in A$, $\hat{x} = F(\mathbf{x}) \in B$, $F: A \rightarrow B$. The problem defined this way is possible to model by supervised methods (if classes $\omega_{i,j} \in \Omega$ of the objects are known) or by unsupervised methods (if classes $\omega_{i,j} \in \Omega$ are not known).

3.1. Model design and data pre-processing

The classes $\omega_{i,j} \in \Omega$ of municipal creditworthiness are not known a priori. Therefore, it is suitable to realize the modelling of municipal creditworthiness by unsupervised methods. Based on the analysis presented in [8], the combination of SOFMs and K-means algorithm is a suitable method for municipal creditworthiness modelling. Its results are used as the inputs of the FLNN [6] in the model presented in Fig. 1.

Supervised learning is realized by the FLNN, which makes the suitable economic interpretation of created clusters possible. Data pre-processing is carried out by means of data standardization. Thereby, the dependency on units is eliminated.

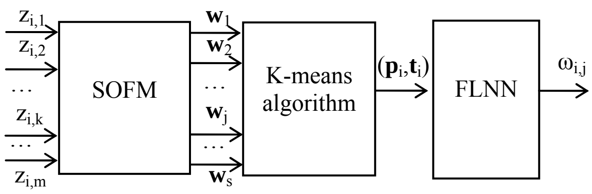

Fig. 1 Model design, where $\mathbf{p}_i = (x_{i,1}, x_{i,2}, \dots, x_{i,k}, \dots, x_{i,m})$, m=12 is the i-th pattern, $z_{i,1}, z_{i,2}, \dots, z_{i,k}, \dots, z_{i,m}$ are standardized values of parameters $x_1, x_2, \dots, x_m$ for the i-th object $o_i \in O$, $(\mathbf{p}_i, \mathbf{t}_i)$ are patterns $\mathbf{p}_i$ assigned to classes $\omega_{i,j} \in \Omega$, $\omega_{i,j} \in \Omega$ is the output of the FLNN.

3.2. Municipal creditworthiness modelling by Kohonen's self-organizing feature maps

Kohonen's self-organizing feature maps [3,4] are based on competitive learning strategy. The input layer serves the distribution of the input patterns $\mathbf{p}_i$, i=1,2, … ,n. The neurons in the competitive layer serve as the representatives (Codebook Vectors), and they are organized into topological structure (most often a two-dimensional grid), which designates the neighbouring network neurons. First, the distances d_j are computed between pattern $\mathbf{p}_i$ and weights of all neurons $\mathbf{w}_{i,j}$ in the competitive layer according to the relation

$$d_j = \sum_{i=1}^{n} (\mathbf{p}_i - \mathbf{w}_{i,j})^2 , \tag{1}$$

where j goes over s neurons of competitive layer, j=1,2, … ,s, $\mathbf{p}_i$ is the i-th pattern, i=1,2, … ,n, $\mathbf{w}_{i,j}$ are synapse weights. The winning neuron j* (Best Matching Unit, BMU) is chosen, for which the distance d_j from the given pattern $\mathbf{p}_i$ is minimum. The output of this neuron is active, while the outputs of other neurons are inactive. The aim of the SOFM learning is to approximate the probability density of the real input vectors $\mathbf{p}_i \in R^n$ by the finite number of representatives $\mathbf{w}_j \in R^n$, where j=1,2, … ,s. When the representatives $\mathbf{w}_j$ are identified, the representative $\mathbf{w}_{j*}$ of the BMU is assigned to each vector $\mathbf{p}_i$. In the learning process of the SOFM, it is necessary to define the concept of neighbourhood function, which determines the range of cooperation among the neurons, i.e. how many representatives $\mathbf{w}_j$ in the neighbourhood of the BMU will be adapted, and to what degree.

Gaussian neighbourhood function is in common use, which is defined as

$$h(j^*, j) = e^{\left(-\frac{d_E^2(j^*,j)}{\lambda^2(t)}\right)}, \qquad (2)$$

where $h(j^*,j)$ is neighbourhood function, $d^2_E(j^*,j)$ is Euclidean distance of neurons j^* and j in the grid, $\lambda(t)$ is the size of the neighbourhood in time t. After the BMUs are found, the adaptation of synapse weights $\mathbf{w}_{i,j}$ follows. The principle of the sequential learning algorithm [4] is the fact, that the representatives $\mathbf{w}_{j*}$ of the BMU and its topological neighbours move towards the actual input vector $\mathbf{p}_i$ according to the relation

$$\mathbf{w}_{i,j}(t+1) = \mathbf{w}_{i,j}(t) + \eta(t)h(j^*, j)(\mathbf{p}_i(t) - \mathbf{w}_{i,j}(t)), \qquad (3)$$

where $\eta(t)\in(0,1)$ is the learning rate. The batch learning algorithm of the SOFM [4] is a variant of the sequential algorithm. The difference consists in the fact that the whole training set O_{train} passes through the SOFM only once, and only then the synapse weights $\mathbf{w}_{i,j}$ are adapted. The adaptation is realized by replacing the representative $\mathbf{w}_j$ with the weighted average of the input vectors $\mathbf{p}_i$ [4].

Using the SOFM as such can detect the structure in the data. The K-means algorithm can be applied to the adapted SOFM in order to find clusters as it is presented in Fig. 2. This way, the topology of the data is preserved and the computational load decreases. Interpretation of clusters is realized by the values of parameters $\mathbf{x}=(x_1,x_2, \dots ,x_m)$, m=12 for individual representatives $\mathbf{w}_j$ (Fig. 3).

The K-means algorithm belongs to non-hierarchical algorithms of cluster analysis, where patterns $\mathbf{p}_1,\mathbf{p}_2, \dots ,\mathbf{p}_i, \dots ,\mathbf{p}_n$ are assigned to clusters $c_1,c_2, \dots ,c_r, \dots ,c_q$. The number of clusters q=6 is determined by indexes evaluating the quality of clustering [1].

Clustering process is realized in two levels. In the first level, n objects are reduced to representatives $\mathbf{w}_1,\mathbf{w}_2, \dots ,\mathbf{w}_s$ by the SOFM, the s representatives are clustered into q=6 clusters.

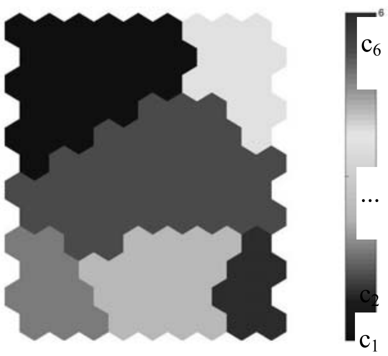

Fig. 2 Clustering of the SOFM by K-means algorithm

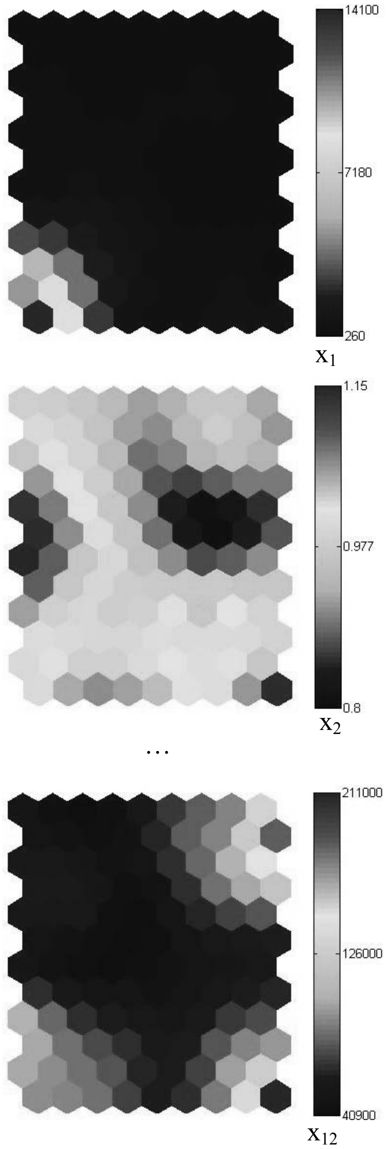

Fig. 3 Values of parameters $x_1,x_2, \dots ,x_m$, m=12 for individual representatives $\mathbf{w}_j$

3.3. Municipal creditworthiness modelling by fuzzy logic neural networks

It is necessary to interpret clusters $c_1,c_2, \dots ,c_q$, q=6 and to label them by classes $\omega_{i,j}\in\Omega$, $\Omega=\{\omega_{1,j},\omega_{2,j}, \dots ,\omega_{i,j}, \dots ,\omega_{n,j}\}$, j=1,2, … ,6 so that the class $\omega_{i,1}$ represents the best municipal creditworthiness, while class $\omega_{i,6}$ represents the worst one. The results of unsupervised learning are used as inputs of the FLNN suitable for classification problem realization [6]. The FLNN parameters make the appropriate interpretation possible. It is a perceptron-type FLNN with three layers. The input layer represents the input parameters $\mathbf{x}=(x_1,x_2, \dots ,x_m)$, m=12 the hidden layer represents rules $R_1,R_2, \dots ,R_u, \dots ,R_N$ and the output layer denotes classes $\omega_{i,j}\in\Omega$. Standard neurons are replaced by fuzzy neurons in the FLNN. Contrary to standard neuron, the synapse weights among fuzzy neurons assign the value of membership function to the input value. The output value μ_y can be defined as

$$\mu_y = \mu_1(x_1) \otimes \mu_2(x_2) \otimes ... \otimes \mu_q(x_q) , \quad (4)$$

where operator $\otimes$ represents one of the fuzzy operations (MIN, MAX) [9]. The membership function μ_y is defined as a continuous function, whose parameters are adapted in the learning process. The FLNN model for classification problem realization is presented in Fig. 4.

The aim of the designed classifier is to achieve a minimum total classification error E_T and maximum classification accuracy S. At the same time, suitable interpretation of the designed model is demanded (i.e. low number N of rules R_u with low number of variables in antecedent, low number k of membership functions, etc.). The classifier is designed in two steps. First, the topology of the FLNN is determined.

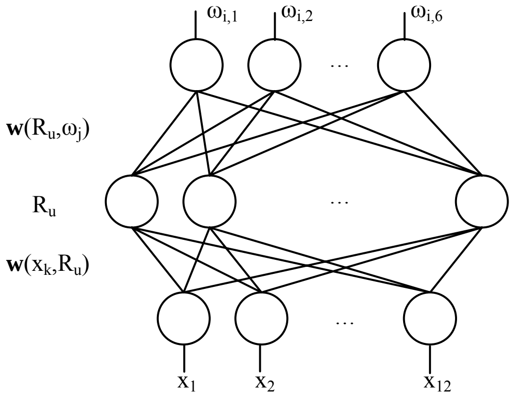

Fig. 4 FLNN model design

Neurons in the hidden layer use t-norms as activation functions, neurons in the output layers use t-conorms [7]. The values of linguistic variable $A_k(i)$ appropriate for the corresponding fuzzy sets are represented by synapse weights $\mathbf{w}(x_k,R_u)$. Synapse weights $\mathbf{w}(R_u,\omega_{i,j})$ represent rules weights R_u. If $\mathbf{w}(R_u,\omega_{i,j})\in[0,1]$, then every neuron in the hidden layer is connected with the only one neuron in the output layer.

Then, parameters of the system are identified in the process of learning. Rules R_u and fuzzy sets represent the parameters. Rules R_u are determined by the following algorithm.

Let FLNN has m inputs $x_1,x_2, \dots ,x_k, \dots ,x_m$, $N<N_{max}$ neurons representing rules $R_1,R_2, \dots ,R_u, \dots ,R_N$ and q output neurons representing classes $\omega_{i,j}\in\Omega$. Next, let $O=\{(\mathbf{p}_1,\mathbf{t}_1),(\mathbf{p}_2,\mathbf{t}_2), \dots ,(\mathbf{p}_i,\mathbf{t}_i), \dots ,(\mathbf{p}_n,\mathbf{t}_n)\}$ is a set of n patterns, where $\mathbf{p}_i\in R^n$ is the vector of parameters values and $\mathbf{t}\in\{0,1\}^q$ is the vector of patterns $\mathbf{p}_i$ assignments to classes $\omega_{i,j}$. Then, the learning algorithm of rules R_u is as follows:

1. Select an object $(\mathbf{p}_i,\mathbf{t}_i)\in O$.
2. $\forall$ inputs x_k find a membership function $\mu^{(k)}_{j_k}(x_k)$ as $\mu^{(k)}_{j_k}(x_k) = \max\limits_{j=\{1,2,\dots,v_k\}} \{\mu^{(k)}_j(x_k)\}$, where v_k is the number of membership functions corresponding to input parameter x_k.
3. If $\exists$ rule R_u with weights $w(x_1,R_u) = \mu^{(1)}_{j_1}$, $w(x_2,R_u) = \mu^{(2)}_{j_2}$, … , $w(x_m,R_u) = \mu^{(m)}_{j_m}$, then create the neuron by rule R_u and connect it with output neuron (class), for which $t_j=1$.
4. If there are objects in set O, which did not pass through the learning algorithm, continue with 1. Else, determine the best N rules R_u and remove all the others rules R_u.

Fuzzy sets are determined by the following algorithm:

1. Select an object $(\mathbf{p}_i,\mathbf{t}_i)\in O$, let it pass through the FLNN and determine $\omega_{i,j}\in\Omega$.
2. $\forall$ classes $\omega_{i,j}$ determine $\Delta_{\omega_{i,j}} = t_j - y_{\omega_{i,j}}$, where $y_{\omega_{i,j}}$ is output of neuron $\omega_{i,j}$.
3. $\forall$ rules R_u with $y_{R_u} > 0$ determine $\Delta_{R_u} = y_{R_u}(1-y_{R_u}) \sum\limits_{\omega_{i,j}\in\Omega} w(R_u,\omega_{i,j})\Delta_{\omega_{i,j}}$, where y_{R_u} is output of neuron R_u.
4. Find such x' as weights $w(x',R_u)y_{x'} = \min\limits_{x_k\in x}\{w(x_k,R_u)y_{x_k}\}$, where y_{x_k} is output of neuron x_k.
5. Determine the parameters of membership functions (e.g. triangular) for fuzzy set represented by synapse weight $\mathbf{w}(x',R_u)$ as follows

$$\Delta_b = \eta\Delta_{R_u}(c-a)\,\mathrm{sgn}(y_{x'} - b) , \quad (5)$$

$$\Delta_a = -\eta\Delta_{R_u}(c-a) + \Delta_b , \quad (6)$$

$$\Delta_c = \eta\Delta_{R_u}(c-a) + \Delta_b , \quad (7)$$

where $\eta > 0$ is learning rate.

6. If the target criterion is reached (total classification error E_T or number of iterations), finish the learning process, else continue with 1.

4. ANALYSIS OF THE RESULTS

Municipal creditworthiness modelling is realized for different numbers and shapes of membership functions and for different numbers N of rules R_u. The set $o_i\in O$, $O=\{o_1,o_2, \dots ,o_i, \dots ,o_n\}$ (n=452 municipalities of Pardubice region, the Czech republic) is divided into training set O_{train} and testing set O_{test}. The quality of classification is measured by average classification error E and classification accuracy S for training and testing data. The results of classification for bell membership functions are presented in Table 2. If the number k of membership functions increases, then the classification accuracy S increases too.

Table 2 The results of classification for bell membership functions, where k is the number of membership functions; S_{train}, S_{test} are the classification accuracies for sets O_{train}, O_{test}, E_{test} is the mean classification error for the set O_{test}, σ_{test} is the standard deviation of the classification accuracy S_{test}.

v_k	S_{train} [%]	S_{test} [%]	E_{test}	σ_{test} [%]	N
2	81.42	80.30	0.39	3.60	23
3	85.62	84.30	0.33	3.46	65
4	87.17	86.74	0.31	3.37	110
5	92.48	88.21	0.27	2.11	174
6	92.48	90.73	0.26	2.86	211
7	95.13	93.30	0.24	2.14	263

A sample of the rules R_u for the number v_k=5 of membership functions is as follows

R_1 : IF x_1 is VS AND x_2 is VS AND x_3 is VS AND x_4 is VS AND x_5 is VS AND x_6 is VS AND x_7 is VS AND x_8 is VS AND x_9 is VS AND x_{10} is VS AND x_{11} is VL AND x_{12} is VS THEN c_1,

R_2 : IF x_1 is VL AND x_2 is VS AND x_3 is VS AND x_4 is VS AND x_5 is VS AND x_6 is VS AND x_7 is M AND x_8 is VS AND x_9 is VS AND x_{10} is VS AND x_{11} is VL AND x_{12} is VS THEN c_1,

...

R_{165} : IF x_1 is VS AND x_2 is VS AND x_3 is VS AND x_4 is VS AND x_5 is VS AND x_6 is VL AND x_7 is VS AND x_8 is M AND x_9 is S AND x_{10} is VS AND x_{11} is VS AND x_{12} is VL THEN c_6,

where VS is a very small, S is a small, M is a medium, L is a large and VL is a very large value of parameter x_k, k=1,2, … ,12. Based on the created rules R_1,R_2, … ,R_N, N=165, clusters c_1,c_2, … ,c_q, q=6, can be labelled by classes $\omega_{i,j}\in\Omega$, where the class $\omega_{i,j}\in\Omega$, j=1 represents the best municipal creditworthiness, while class $\omega_{i,j}\in\Omega$, j=q, represents the worst one. This process has to be realized by an expert from the field of municipal creditworthiness evaluation. Then, the classes $\omega_{i,j}\in\Omega$, j=1,2, … ,6, can be characterized as presented in Table 3.

Table 3 Descriptions of classes $\omega_{i,j}\in\Omega$

Class $\omega_{i,j}$ j=1,2, … ,6	Description
$\omega_{i,1}$	High ability of a municipality to meet its financial obligation. Very favourable economic conditions, low debt and excellent budget implementation.
$\omega_{i,2}$	Very good ability of a municipality to meet its financial obligation.
$\omega_{i,3}$	Good ability of a municipality to meet its financial obligation.
$\omega_{i,4}$	A municipality with stable economy, medium debt and good budget implementation.
$\omega_{i,5}$	Municipality meets its financial obligation only under favourable economic conditions.
$\omega_{i,6}$	A municipality meets its financial obligations with difficulty, the municipality is highly indebted.

The suitability of the designed model can be compared with other fuzzy logic based classifiers as presented in Table 4.

Table 4 Comparison of the FLNN model with the maximum support method (MSM) and fuzzy decision trees (FDTs) classifiers in terms of classification accuracy S.

	MSM		FDT		FLNS	
v_k	S_{train}	S_{test}	S_{train}	S_{test}	S_{train}	S_{test}
2	67.26	54.42	66.81	58.41	81.42	80.30
3	88.50	55.04	75.66	66.37	85.62	84.30
4	96.46	56.64	84.07	67.70	87.17	86.74
5	98.23	45.13	85.84	65.49	92.48	88.21
6	99.56	34.07	85.40	65.49	92.48	90.73
7	99.12	15.49	91.15	64.60	95.13	93.30

Fig. 5 represents the classification of municipalities into classes $\omega_{i,j}\in\Omega$. The left parts of the histogram are the results of the SOFM and the right parts are the results of the FLNN. The results stand for absolute frequencies of municipalities f_m in the j-th class $\omega_{i,j}\in\Omega$.

Fuzzy IF-THEN classifiers [5] MSM and FDT are selected. The input-output membership functions for fuzzy sets and their universes are designed for classifiers MSM and FDT by the K-means algorithm. The MSM method is better than the FLNN model in terms of classification accuracy S_{train}. The highest classification accuracy S_{test} is achieved by the FLNN. The FLNN model is the best in terms of generalization ability due to this fact.

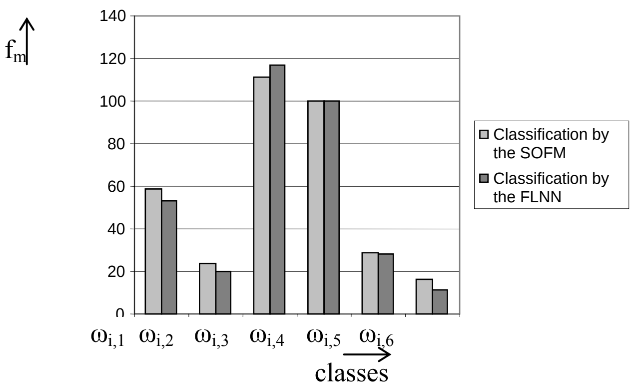

Fig. 5 Classification of municipalities into classes (v_k=5).

5. CONCLUSION

The paper presents the design of municipal creditworthiness parameters. Further, the model is designed whereby municipal creditworthiness evaluation is realized. First, the data are pre-processed. Next, municipal creditworthiness modelling is realized by the SOFM. The outputs of the SOFM are used as the inputs of the FLNN. Its aim is the classification of municipalities into classes so that high classification accuracy, low average classification error and suitable interpretation of results are achieved. The results are compared with other fuzzy logic based classifiers.

The designed model was carried out in NefClass (FLNN) and Matlab (SOFM) in MS Windows XP operation system.

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BIOGRAPHIES

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