ANT COLONY OPTIMIZATION TO SHUNT CAPACITOR ALLOCATION IN RADIAL DISTRIBUTION SYSTEMS

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SUMMARY

This paper presents an ant colony approach optimization to shunt capacitor placement on distribution systems under capacitor switching constraints. The optimum capacitor allocation solution is found for the system of feeders fed through their transformer and not for any individual feeder. The main advantages due to capacitor installation, such as system capacity release and reduction of overall power and energy losses are considered. The capacitor allocation constraints due to capacitor-switching transients are taken into account. These constraints are extremely important if pole-mounted capacitors are used together with a station capacitor bank. An ant colony algorithm is used as an optimization tool. An illustrative example for Algerian network is presented.

Keywords: *Distribution system; Capacitor allocation; Ant colony algorithm, optimisation*

Nomonclature:
ALDC: Annual Load Demand Curve.
ACO : Ant Colony Optimization.
EGA : Electrical And Gaz Algerian.
ACP : Automatic Changing Plot.
SCR : System Capacity Release.
M.DA: Mega Algerian Dinars

1. INTRODUCTION

Capacitors are widely used in distribution systems for reactive power compensation to achieve power and energy loss reduction, system capacity release and acceptable voltage profile. The extent of these advantages depends on the location, size, type and number of the capacitors as well as on their control settings. The capacitor allocation problem is a well-researched topic. Actually at the early stage of research, the main advantages that can be derived from applying shunt capacitors to the distribution feeders were evaluated. The problem was solved using analytical methods for simplified models of the feeders and their load distribution Ref [1–2–3–5–6]. The need to find a general solution for real distribution systems together with advances in computer technology brought to life a new generation of methods and techniques based on computer applications in Ref [7-8-9]. These methods presented the annual return that can be yielded from capacitor application as an explicit function of location and size of pole-mounted capacitors. Different numerical methods were used to maximize the return and to find optimum capacitor allocation. All of the above mentioned approaches considered capacitor application to an individual feeder. In Ref [10-11-12-13] the authors incorporated the station capacity release into their models. In Ref [11] the evaluation of energy losses in a station transformer was included. It should be noted that a station capacity release and losses reduction in the transformer could not be calculated based on capacitor allocation on an individual feeder. These values should be determined by the station capacitor bank (if any). Within the past ten years further research has taken place in the field of optimal capacitor placement in Ref [14-15]. Load-flow on balanced or unbalanced radial feeders has been employed to evaluate the fitness of arbitrary solutions. Optimum capacitor placement has been achieved using sophisticated methods such as fuzzy logic, simulated annealing and genetic algorithms recentelly the ant colony algorithm (*ACO*) will be applied to this problem. The above-mentioned methods have been used for capacitor allocation on an individual feeder, while only real-power losses in the feeder are considered. These methods enable one to find the optimum capacitor allocation and control solution for an individual feeder from the viewpoint of loss reduction in the feeder. Since overall system advantages due to the capacitor application are not considered, this solution cannot be treated as an optimal one. Besides, determining capacitor placement from the standpoint of individual feeders and neglecting capacitor switching transients may lead to inserting a pole-mounted capacitor close to a station capacitor bank or to other pole-mounted capacitors. Switching a pole-mounted capacitor in the vicinity of other capacitors may subject the capacitor to extremely high inrush currents, which cause failures of capacitors and their switch-gear.

The primary objective of this work is to present an ant colony approach to the problem of capacitor allocation on radial distribution feeders. Capacitor allocation is treated as a medium voltage (MT) reactive power planning procedure, while special concern is given to constraints imposed by capacitor switching transients. An ant colony algorithm is used to determine optimal placement and control of capacitors, so that the economic advantages achieved from system capacity release, overall peakload power and energy losses reduction is maximized.

The rest of this paper is outlined as follows. We start in section 2 with the problem formulation and system model description. Next, we describe the advantage of capacitor implementation in section 3. The Capacitor allocation constraints is presented in section 4. In section 5, we present an illustrative example and conclusion.

2. PROBLEM FORMULATION AND SYSTEM MODEL DESCRIPTION

In the general case, reactive power compensation on the medium voltage level can be achieved by the combination of a station capacitor bank on the secondary side of the station transformer with pole-mounted capacitors on the downstream distribution feeders. Main economic advantages that can be derived from medium voltage capacitor application can be summarized in Ref [1-2] and [16] as:

(i) Transmission and transformation kVA capacity release.

(ii) Reduction of overall system peak-load losses.

(ii) Reduction of annual system energy losses.

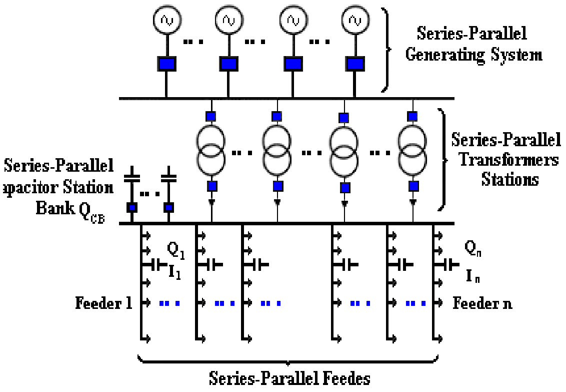

Fig. 1 Series-parallel distribution system model for capacitor allocation

The system approach to optimal capacitor allocation on the distribution level is to determine kVAr ratings, placement and control settings of a station capacitor bank and of pole-mounted capacitors that maximize the above mentioned system advantages against the cost of capacitors. The optimal capacitor allocation solution shall also meet the requirements of an acceptable voltage profile along feeders in peak-load and *off-peak* states and shall conform to permissible inrush currents during capacitor switching. Optimal capacitor allocation is considered for the general series-parallel distribution system model as shown in Fig.1. Loads are supplied through radial distribution feeders fed from one of station transformers. Since parallel operation of station transformers is generally used, an equivalent scheme of power supply to some distribution system can be presented as a series connection of a transmission system, a station transformer and a network of feeders. The transmission system is presented by a serie-parallel network. The transformer model is comprised of an excitation branch and of transformer series impedance connected in series with an ideal transformer. Since most transformers are equipped with ACP, the transformation factor is assumed to be changing with transformer loading, so as to keep constant (or load-dependent) voltage on the secondary side of the transformer. The distribution system is comprised of a network of radial distribution feeders. Each feeder includes a three-phase symmetrical main feeder and three-phase symmetrical lateral branches having any configuration, any conductor sizes and any number of distribution transformers of different kVA rating. Loads are treated as constant power sinks. Shunt capacitors are represented as susceptances, whose reactive power injection is proportional to the square of voltage at their nodes. The advantages can be calculated based on load-flows in the considered system model. Since we are interested in thermal capacity release and peak-load loss reduction, the readings of peak-load loading of the feeders, of the feeding transformer and of the station are required. Calculation of annual energy loss reduction necessitates using Annual Load Duration Curves of the whole station and of the considered transformer. These curves are approximated by piecewise linear functions. The year is divided into n intervals during which the load profiles and load distribution between feeders are assumed to be constant as in Ref [16-17]. We suppose that loads on each feeder vary in a conformal way proportional to annual load demand curve of the feeding transformer and to the power (current) loading of the feeder. For each load level, power flow calculations are performed to determine power losses and voltage variations along the feeders. Optimum capacitor placement on a distribution system includes optimum allocation of pole-mounted capacitors in addition to installation of a station capacitor bank on the secondary side of the feeding transformer. We assume that due to standardization, utilities generally use one or two sizes of station capacitor banks on the secondary side of a specific transformer and two or three sizes of pole-mounted capacitors. For a specified rating of a station capacitor.

3. ADVANTAGE OF CAPACITOR IMPLEMENTATION

We consider the series-parallel distribution system as depickted in Fig. 1 which has the station capacitor bank with Q_{CB} kVAr rating and k pole-mounted capacitors arbitrarily allocated on the downstream feeders. Each j-th pole-mounted capacitor is characterized by its kVAr rating Q_j belonging to a specified set of capacitor ratings, by its cost C_j and by its location $\left(f_j, l_j\right)$, where f_j is the number of a feeder and l_j is the number of the section on which the capacitor is installed.

3.1. Advantage of System Capacity

Unlike calculations of system capacity release in Ref [10], [11] and [13], the proposed method takes into account parameters of the station transformer and reduction of active and reactive power losses in the distribution system and in the feeding transformer due to application of medium voltage capacitors. The primary loading of a transformer, S_1 can be expressed as a function of its secondary load $S_2 = P_2 + jQ_2$

$$S_1 = \frac{1}{V^2}\left[\left(P + R_{tr}\left(P^2 + q^2\right)\right)^2 + \left(q + X_{tr}\left(P^2 + q^2\right)\right)^2\right]^{\frac{1}{2}} \quad (1)$$

where:

$$P = P_2 + \Delta P_{FE}\left(\frac{V}{V_n}\right)^2 \quad And \quad q = Q_2 + \Delta Q_{FE}\left(\frac{V}{V_n}\right)^2$$

In Eq. (1):

R_{tr}, X_{tr} : Are the transformer series resistance and leakage reactance.

$\Delta P_{FE}, \Delta Q_{FE}$: Are the no-load transformer active and reactive losses, respectively.

V : Is a voltage maintained on the secondary side of the transformer.

V_n: Is the rated secondary voltage of the transformer. Under the peakload conditions without reactive power compensation the transformer loading on the primary side is $S_{1max} = f\left(P_{2max}, Q_{2max}\right)$, where P_{2max} and Q_{2max} are the maximum active and reactive load, respectively on the secondary side. The application of the station capacitor bank and pole-mounted capacitors results in the reduction of peak-load active power losses in the distribution system by ΔP_c and in the total reduction of reactive power on the secondary side of the transformer by ΔQ_c. Thus, the additional load $g\left(P_{2max}, Q_{2max}\right)$ can be served by the transformer without increasing its primary side loading $S_{1\max}$. The per-unit load increase g can be determined from the equation:

$$S_{1max} = f\left(\left(P_{2max}(1+g) - \Delta p_c\right), \left(Q_{2max}(1+g) - \Delta Q_c\right)\right) \quad (2)$$

The system capacity release ***SCR*** can be expressed as a function of g as follows:

$$\Delta S_{CR} = f\left(P_{2max}(1+g), Q_{2max}(1+g)\right) - S_{1max} \qquad (3)$$

The annual benefit due to the released system capacity is $C_S \Delta S_{CR}$, where C_S is the cost of system thermal capacity release.

3.2. Advantage of Peak-Load Loss Reduction

The cost of peak-load loss reduction C_{PL} can be represented as:

$$C_{PL} = \left(P_{L\,max} - P_{comp}^{L\,max}\right)c_L \qquad (4)$$

Where $P_{L\,max}$ is system peak-load losses, which include losses in the distribution system, in its transformer and in the equivalent transmission system without reactive power compensation; $P_{comp}^{L\,max}$ is the same when the considered medium voltage capacitors are applied; c_L is annual cost of peak-load real power losses $\left[\$/kW\right]$ or $\left[DA/kW\right]$.

One \$ USA : Bought ***100*** Dinar Algerian

3.3. Advantage of Energy Loss Reduction

Annual energy loss reduction depends on control of the considered capacitors during the year. It is assumed that the station capacitor bank energization is controlled by its reactive power regulator, while the pole-mounted capacitors are switched by their time-control devices. In order to evaluate the fitness of any arbitrary solution, it is assumed that all pole-mounted capacitors are switched in conforming way at every load level. Therefore, for each load level the following possible modes of capacitors are considered:

1. All medium voltage capacitors are disconnected.
2. All pole-mounted capacitors are switched on, while a station capacitor bank is disconnected.
3. All pole-mounted capacitors as well as a station capacitor bank are in operation. The assumption of the conforming control of pole-mounted capacitors is suitable for distribution systems with conformal variations of their loads, if the time controlled capacitors of **20** kVAr ratings and higher are used. To determine the optimum capacitor control resulting in maximum energy reduction at i-th load level, total power loss reduction for all the above-mentioned modes of capacitor operation should be calculated.

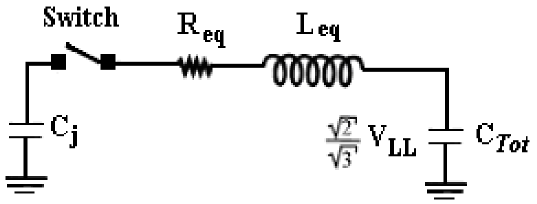

Fig. 2 Equivalent circuit for calculation of capacitor switching transients system

The maximum power loss reduction at i th load level is calculated as follow:

$$\Delta P_{L_i}^{\max} = \max\left\{0, P_{Li}^{i}\right\} \qquad (5)$$

Where, P_{Li}^{i} is the total system loss at i th load level.

3.4. Capacitor Switching Constraint

A pole-mounted capacitor shall be allocated in such a way as to prevent high inrush currents caused by its interaction with other capacitors on the distribution system. The peak value of the inrush current in pole-mounted capacitor *j* shall be less than the magnitude $I_{max\,j}$ determined by the acceptable value of peak current for its capacitors in Ref [18] or switchgear. The $I_2\ (t)$ duty of a transient current in the pole-mounted capacitor due to its energization shall not exceed the maximum value. Cost of energy loss reduction at *i*-th load level is determined as follows:

$$C_{PL} = C_E \times \sum_{i=1}^{n} \Delta P_{Li}^{\max} \times T_i \tag{6}$$

where

C_E represent the cost of energy losses.

T_i represent time duration of load level (*i*).

3.5. Annual Return

The annual return obtained from the capacitor application C_{Tot} can be presented by equations (4) and (6) as:

$$C_{Total} = C_{CR} + C_{PL} + C_{EL} + C_{CB} - \sum_{j=1}^{k} C_j \tag{7}$$

where C_{CB} is annual cost of the station capacitor bank; C_{PL} is the cost energy loss reduction and C_j capacitor *j*. The main system advantages due to capacitors' application, as was shown above, depend on kVAr ratings Q_j, location l_j and control of the pole-mounted capacitors. So, if the MVAr of the station capacitor bank is specified, the optimal allocation and control of pole-mounted capacitors can achieve the maximum return.

4. CAPACITOR ALLOCATIONS CONSTRAINTS

Determining sizes and locations of pole-mounted capacitors requires taking into account constraints imposed by voltage variation at the load nodes and by capacitor switching transients.

♦ ***Voltage Constraint***: The voltage constraints can be taken into account by specifying upper and lower limits of voltage variation at the nodes of the distribution system [16-17]. For every node *m* in the distribution system at every load level *i*, these constraints can be expressed as

$$V_{min}^2 \le V_{mi}^2 \le V_{max}^2 .m = 1,\ldots,M, i = 1,\ldots,n \tag{8}$$

where *M* is total number of nodes in the distribution system. $V_{\min}^2$, V_{mi}^2, $V_{\max}^2$ represents minimum, midium and maximum voltage.

♦ ***Capacitor Switching Constraint:*** A pole-mounted capacitor shall be allocated in such a way as to prevent high inrush currents caused by its interaction with other capacitors on the distribution system. The peak value of the inrush current in pole-mounted capacitor *j* shall be less than the magnitude $I_{\max j}$ determined by the acceptable value of peak current for its capacitors [18] or switchgear. The $I^2(t)$ duty of a transient current in the pole-mounted capacitor due to its energization shall not exceed the maximum value $I^2(t_{\max})$ that the fuses can withstand without spurious melting [17].

In general, determining the transient switching current of a capacitor energized in a distribution system which contains other capacitors requires use of the Electromagnetic Transient Program Ref [19]. To impose the current switching constraints on capacitor allocation, we propose a simplified analytical method to determine the peak switching current and its $I_2(t)$ value. This method is based on the assumption that during switching of a capacitor all other capacitors already consedired as a single capacitor in Ref [19]. Therefore, the capacitances of the already switched capacitors can be lumped together. The switching current of pole-mounted capacitor, due to its interaction with other capacitors, can be calculated using the equivalent circuit of capacitor back-to-back switching in Ref [20] refer to Fig. 2. Back-to-back switching current $I_{SW}(t)$, which results from switching a capacitor with capacitance C_j against the equivalent system capacitor with capacitance C_{Tot}, can be expressed as:

$$\begin{cases} I_{SW}(t) = \dfrac{\sqrt{2}}{\sqrt{3}} .V_{LL} \times \sqrt{\dfrac{C_{eq}}{L_{eq}}} .e^{\frac{-R_{eq}}{L_{eq}}t} .\sin \dfrac{1}{\sqrt{L_{eq} \times C_{eq}}} t \\ IF \quad \dfrac{L_{eq}}{C_{eq}} \ge\ge \dfrac{R_{eq}^2}{4} \end{cases} \tag{9}$$

Where $C_{eq} = \frac{c_j .C_{Tot} C_{eq}}{C_j + C_{tot}}$ is equivalent capacitance of the circuit; R_{eq} and L_{eq} are alternating current resistance and inductance, respectively, between the capacitor being energized and the capacitor already energized; V_{LL} is phase-to-phase maximum system voltage. The peak value of the inrush current $I_{SW\,max}$ and its $I^2(t)$ value can be expressed in terms of rated reactive power of the considered capacitor Q_j, rated reactive power of the corresponding equivalent system capacitor Q_{tot} and their mutual impedance:

$$\begin{cases} I_{SW\,\max j} = \dfrac{\sqrt{2}}{\sqrt{3}}.V_{LL}.\sqrt{\dfrac{C_{eq}}{L_{eq}}} = \sqrt{\dfrac{2}{3}.\dfrac{Q_j.Q_{tot}}{\left(Q_j + Q_{tot}\right).X_{eq}}} \\ I^2 t_j = \displaystyle\int_0^\infty I_{SW}^2(t)dt = \dfrac{Q_j.Q_{tot}}{3.\left(Q_j + Q_{tot}\right).R_{eq}.\omega} \end{cases} \tag{10}$$

where $X_{eq} = \omega.L_{eq}$ is equivalent reactance between the capacitors and V is fundamental angular frequency. Therefore, the capacitor switching constraints imposed on the allocation of pole-mounted capacitor j can be presented as

$$I_{SW\,max\,j} \leq I_{max\,j} I^2 t_{S\,max\,j} \leq I^2 t_{max\,j} \tag{11}$$

Imposing the switching constraints on allocation of every pole-mounted capacitor requires calculation of parameters of the equivalent circuit of its switching: Q_{tot}, R_{eq}, X_{eq}. The calculation of the above parameters is achieved by circuit reduction with reference to the node of the considered pole-mounted capacitor. The station capacitor bank presents an equivalent capacitor on the station bus bars. This capacitor in its turn is paralleled with other capacitors on the feeder, where the considered capacitor is installed. Paralleling branches with capacitors implies lumping together their capacitances along with connecting their impedances in parallel.

5. ANT COLONY ALGORITHM APPROACH

Many researchers have shown that insect colonies behavior can be seen as a natural model of collective problem solving. The analogy between the way ants look for food and combinatorial optimization problems has given rise to a new computational paradigm, which is called ant colony meta-heuristic. This paper presents an application of ant colony in shunt capacitor allocation in radial distribution systems optimization problem.

Ants lay down in some quantity an aromatic substance, known as *pheromone*, in their way to food. The pheromone quantity depends on the length of the path and the quality of the discovered food source. An ant chooses a specific path in correlation with the intensity of the pheromone. The pheromone trail evaporates over time if no more pheromone is laid down. Other ants can observe the pheromone trail and are attracted to follow it. Thus, the path will be marked again and will therefore attract more ants. The pheromone trail on paths leading to rich food sources close to the nest will be more frequented and will therefore grow faster. In that way, the best solution has more intensive pheromone and higher probability to be chosen. The described behaviour of real ant colonies can be used to solve combinatorial optimization problems by simulation: artificial ants searching the solution space simulate real ants searching their environment. The objective values correspond to the quality of the food sources. The ant system approach associates pheromone trails to features of the solutions of a combinatorial problem, which can be seen as a kind of adaptive memory of the previous solutions. In order to demonstrate the ant system approach, in Ref [21] apply it to the classical traveling salesman problem, asymmetric traveling salesman problem, quadratic assignment problem, and job-shop scheduling. Ant system shows very good results in each applied area. The ant system has also been applied with success to other combinatorial optimization problems. The ant system method has not yet been used neither in allocation of shunt capacitor problem.

5.1. Overview of the Proposed Method

To apply the ant system (AS) algorithm to a combinatorial optimization problem, it is convenient to represent the problem by a graph G = (ς, **Λ**); where ς are the nodes and **Λ** is the set of edges. To represent our problem as such a graph, the set of nodes ς is given by components (capacitors and size), and edges connect each component to its available size and allocation (length). Some nodes are added to represent positions where additional component was not used. As in Ref [22] these nodes are called blanks nodes and have attributes of zero. The obtained graph is partially connected. Ants cooperate by using indirect form of communication mediated by pheromone they deposit on the edges of the graph G while building solutions.

Informally, our algorithm works as follows: m ants are initially positioned on a node representing a component. Each ant represents one possible solution of the entire system. This solution is represented (f_j, l_j). The (f_j, l_j) can be chosen in any combination from the available type and size of capacitors. Each ant builds a feasible solution (called a tour) to the problem by repeatedly applying a stochastic greedy rule, i.e., the *state transition rule.*

Ants use problem-specific heuristic information (denoted by η_{ij}) and pheromone trails (denoted by τ_{ij}) to select optimal allocation capacitors. An ant positioned on node i chooses the capacitor j by applying the rule given by:

$$J = \begin{cases} \arg \max\limits_{m \in AC_i} ([\tau_{im}]^\alpha [\eta_{im}]^\beta) & if \;\; q \leq q_o \\ J & Otherwise \end{cases} \tag{12}$$

and j is a random variable selected according to the probability distribution given by:

$$p_{ij} = \begin{cases} \dfrac{\left[\tau_{ij}\right]^\alpha \left[\eta_{ij}\right]^\beta}{\sum\limits_{m \in AC_i} \left[\tau_{im}\right]^\alpha \left[\eta_{im}\right]^\beta} & if \;\; j \in AC_i \\ 0 & Otherwise \end{cases} \tag{13}$$

α and β are parameters that control the relative weight of the pheromone. *AC*$_i$ is the set of available components or capacitors.

While constructing its solution, an ant also modifies the amount of pheromone on the visited edges by applying the *local updating rule*:

While building a solution of the problem, ants choose elements by visiting edges on the graph *G,* and change their pheromone level by applying the following local updating rule:

$$\tau_{ij}^{new} = (1-\rho)\tau_{ij}^{old} + \rho\tau_o \qquad (14)$$

where ρ is a coefficient such that (1-ρ) represents the evaporation of trail and τ_0 represent the initial trail of pheromone.

Once all ants have terminated their tour, the amount of pheromone on edges is modified again (by applying the *global updating rule)*:

Once all ants have built a complete system, pheromone trails are updated. Only the globally best ant (i.e., the ant which constructed the best solution from the beginning of the trial) is allowed to deposit pheromone. A quantity of pheromone $\Delta\tau_{ij}$ is deposited on each edge that the best ant has used. Therefore, the global updating rule is:

$$\tau_{ij}^{new} = (1-\rho)\tau_{ij}^{old} + \rho\Delta\tau_{ij} \qquad (15)$$

where $0 < \rho < 1$ is the pheromone decay parameter representing the evaporation of trail and $\Delta\tau_{ij}$ represente the lay of the pheromone in the adge (*i,j*). Ants are guided, in building their tours, by both heuristic information (they prefer to choose "less expansive" edges), and by pheromone information. Naturally, an edge with a high amount of pheromone is a very desirable choice. The pheromone updating rules are designed so that they tend to give more pheromone to edges which should be visited by ants.

6. OVER VIEW ON THE ANT ALGORITHM

STEP. 1 Initialization

Set: NC=0 /*NC : Cycle counter*/.

For every combination (i,j) /*i: Fedeer indice j: Bus_bar indice*/.

Set an initial value and $\Delta\tau_0$

End

For every combination (i,j) of feeders

Set an initial value and

End

Set: $f = 0$ /* f : Possible capacitor location on $k_{\max}$ position*/.

For $f := 0$ to N_f { N_f is the possible number capacitor on fedeer}

Set: $k_{\max}$:0 /* $k_{\max}$: Is the preliminary specified maximal allowable number of capacitors per feeder */.

Set: r_j :0 /* r_j : Type of capacitor and its location */. For every combination (j) compute the:

$$l_j \le r_j \quad if \quad \frac{l_j}{N_f} \ge 0$$

STEP. 2 The average total number of capacitors in the system is:

For i:=1 to N /*N: number of feeders*/.

For j:=1 to $k_{\max}$ /* $k_{\max}$: maximum number capacitors allowed in feeders */.

$$e = K_{\max} \sum_{f=1}^{K_{\max}} \left(\frac{N_f}{KN_C} \right)$$

Or the deserable number of capacitors in the system is given by:

$$e = \frac{(Q_{1\max} - Q_{CB})}{Q_{avg}}$$

Where : Q_{avg} : /* is the average kVAr rating of pole-mounted capacitor*/.

STEP. 3 Choose a capacitors $r(i, j)$ with transition probability given by equations (12-13) :

$$J = \begin{cases} \arg \max\limits_{m \in AC_i} ([\tau_{im}]^{\alpha} [\eta_{im}]^{\beta}) & if \quad q \le q_o \\ J & Otherwise \end{cases}$$

$$p_{ij} = \begin{cases} \dfrac{[\tau_{ij}]^{\alpha} [\eta_{ij}]^{\beta}}{\sum\limits_{m \in AC_i} [\tau_{im}]^{\alpha} [\eta_{im}]^{\beta}} & if \quad j \in AC_i \\ 0 & Otherwise \end{cases}$$

/* This selection can return blanks: No capacitor selected*/ .

STEP. 4 Set According Equ.(7):

$$C_{Tot} = C_{CR} + C_{PL} + C_{EL} + C_{CB} - \sum_{j=1}^{k} C_j$$

Compute the annual cost /* Peak-Load Reduction and Capacirors Constraints*/ .

STEP. 5 Compute:

$$\Omega = C_{Total} + \sum_{m=1}^{M} \sum_{i=1}^{n} \Psi\left(V_{mi}^2 - V_{\max}^2\right) + \sum_{m=1}^{M} \sum_{i=1}^{n} \Psi\left(V_{\min}^2 - V_{mi}^2\right)$$

V_{mi}^2 Represent the medium voltage between the minimum and maximum voltage

$$+ \sum_{j=1}^{k} \Psi\left(I_{SW\max j} - I_{mi}\right) + \sum_{j=1}^{k} \mathrm{B}\Psi\left(I^2(t_j) - I^2(t_{\max j})\right)$$

With: $\psi(x) = \begin{cases} \psi_0(x+1), x > 0 \\ 0, x \le 0 \end{cases}$

STEP. 6 For i:=1 to N /*N: number of feeders*/

For j:=1 to $k_{\max}$ /* $k_{\max}$: maximum number capacitors allowed in feeders */.

```
Local pheromone updating of components to choose according to equation (14) applied to system selection
End. Else.
For i:=1 to N /*N: number of feeders*/
For j:=1 to $k_{\max}$ /* $k_{\max}$: maximum number capacitors allowed in feeders */.
Global pheromone updating of components to choose according to equation (15) applied to system selection
End.

STEP. 7 If {NC<NC} and { not satgnation behaviour}
Then GoTo Step 4.
Else Print and save the best feasible solution.
End.
STOP
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7. ILLUSTRATIVE EXAMPLE

The ant colony algorithm program developed on the base of the proposed new meta-heristic algorithm has been successfully used for optimal allocation of about **25** pole-mounted capacitors in **09** distribution systems of ***ALGERIAN COMPANY OF ELECTRICITY AND GAZ*** (*EGA*). The optimum capacitor allocation and control have been determined for real distribution system models including **50** of loads supplied through radial distribution feeders of real configuration. The real *ALDC* of the step-down transformers have been used. In order to give clear illustration of the proposed method consider optimal capacitor allocation on the simplified distribution system fed by the **90/30 KV** and **3*30** MVA station transformer see Fig.3. The transformer is installed on a station with three uniformly loaded transformers. Loading of each transformer in peak-load condition is $S_{1max} = 32.5\ MW + j17.9\ MVAr$, i.e. ***80%*** of the transformer MVA rating. According to EGA reliability requirements to power supply, the peak-load loading of a transformer on a three-transformer substation shall not exceed ***85%*** of the transformer **MVA** rating. Therefore, the further station load growth will require construction of an additional transformer substation and high voltage transmission lines to connect it to the system grid. The construction of a new substation can be deferred by applying medium voltage power capacitors. Types of **30** kV capacitors to be used for reactive power compensation and their annual costs are listed in Table 1. It is assumed that the both transformers have similar *ALDC* presented as the following function of time *t*:

$$AC(t) = 1 - 0.34.t + 0.154043.t^2 - 0.03635t^3 + .. + t^5$$

The year is divided into 20 equal intervals. Apparent power on the primary side of the considered transformer during interval *i* S_{1i} is assumed to be constant and is expressed as:

$$S_{1i} = 0.5.S_{1max}.\left(AC(t_{ib}) + AC(t_{ie})\right)$$

where t_{ib} and t_{ie} are the beginning and the end of the interval *i*. The distribution network considered is presented in Fig.3. It is assumed that the load is uniformly distributed between four feeders. To simplify system description and at the same time some general regularities in capacitor allocation, each feeder is presented as the combination of uniformly distributed load with end concentrated load. Each feeder includes **50** load nodes separated from each other by uniform feeder sections of **200 m** and total length of each feeder is **17** km. Specific impedance of the feeders is $Z_0 = 0.22 - j0.34\ \Omega.km$. The considered transformers has the following parameters:

- $R_{tr} = 0.0046\ PU$
- $X_{tr} = 0.1842\ PU$
- $\Delta P_{FE} = 29.3\ kW$
- $\Delta Q_{FE} = 42.3\ kVAr$

- The voltage regulator changes the transformation factor with the load variation so as to keep secondary voltage $V_2 = \mp 23.6\ kV$.

- Equivalent system resistance of the transmission system is: $R_{sys} = 2.63\ \ \Omega$ corresponding to 90 kV.

- The annual system costs are:
  - $c_{CR} = 28.5\ \ \$/kW$
  - $c_L = 42.6\ \ \$/kW$
  - $c_E = 0.03\ \ \$/kWh$

First, optimal capacitor allocation was determined regardless of the capacitor switching constraints. The optimum solution includes ***five*** pole-mounted capacitors of **1800** kVAr rating to be installed in addition to the station capacitor bank:

- Four pole-mounted capacitors are uniformly divided among the feeders, so that each capacitor is connected to load bus **50** of the corresponding feeder **9,6** km from the station.

- **One** pole-mounted capacitor is connected to the station bus-bars in parallel to the substation capacitor bank. Switching of the latter pole-mounted capacitor against the station capacitor bank already energized will cause failures of its capacitors together with spurious melting of its fuses.

To determine optimal capacitor allocation with regard to the capacitor switching constraints the acceptable values of the pole-mounted capacitors I_{max} and $I_2(t_{\max})$ were calculated (see Table 1). The optimum capacitor allocation solution includes **five** pole-mounted capacitors of 20 kVAr rating to be installed in addition to the substation capacitor bank (see Fig. 3). Like in the previous solution, **four** polemounted capacitors are connected to **50** load busses of the corresponding feeders. An additional capacitor is connected to load bus **five** of the first feeder. To maximize energy loss reduction, the pole-mounted capacitors shall be in operation throughout the year, while the station bank shall be energized only **8760** h in the year. The results of capacitor allocation can be explained as follows:

Placement of **20** kVAr capacitors at bus **50** of the considered feeders provides maximum peakpower and energy loss reduction in the feeders. On the other hand, system thermal capacity release, peak-power and energy loss reduction in the feeders. Maximum system benefits are yielded if the station capacitor bank and additional **16** kVAr capacitor are placed at the station bus-bars. In order to comply with the capacitor switching constraints, some impedance shall be inserted between the capacitor bank and the pole-mounted capacitor. Placing the capacitor on one of the feeders at **1** km from the station bus-bars. The effectiveness of the capacitor allocation solution presented in Fig. 3 is shown in Table 2.

7.1. Description of the System to be Optimized

The electrical power station system with optimal shunt capacitor allocation which supplies the consumers is designed with 3 basic sub-systems. The detailed process of the electrical power system (production system, transformers system and medium voltage fedeers distribution). The process of electrical power system is describe as follows : The electrical power is generated from the station generators (sub-system 1). Then transformed for medum voltage by the medium voltage transformers (sub-system 2) and distributed by medium voltage fedeers (sub-system 3) which supplies the medium voltage load. A bank of componsation capacitors will be installed in the medium bus and on fedeers.

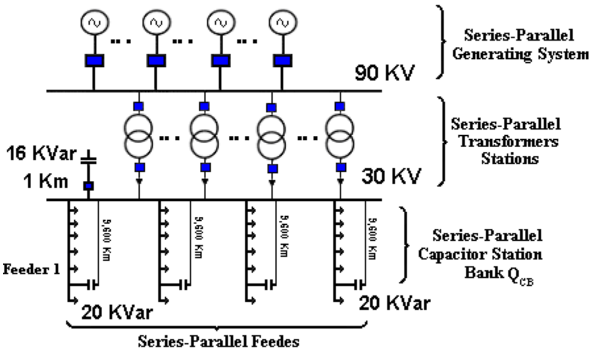

Fig. 3 Optimal Capacitors Allocation

7.2. Optimal Solution Given By The Ant Colony Algorithm

One of the most important problems in power system is the optimal capacitors allocation.

The program developed in this paper is based on the proposed algorithm has been successfully used for optimal shunt capacitor allocation of about **25** pole-mounted capacitors in **09** distribution systems of Ouest Algerian Network. The optimum capacitor allocation and control have been determined for real distribution system models including **50** of loads supplied through radial distribution feeders of optimal configuration. In order to give clear illustration of the proposed method consider optimal capacitor allocation on the simplified distribution system fed by the **90/30** kV station transformer see Fig. 3. Table 1 shows the parametre of annual load demand curve.

# Of Load	50 Bus_Bar
Demand (%)	**98 %**
Duration (h)	**6780**

Table 1 Parameters of annual load demand curve

Table 2 shows the total reduction peak-load loss in the system, capacity cost and annual save. In the description by applying ant colony algorith we determine the optimal allocation capacitor downstream of the transformers and in the fedeers. The program is done in Java and the time to find the optimal solution is about 1mn and 20 second.

Item	Amount	Annual Cost M.DA
System Capacity Release	8 MVA	0.872.200
Peak-Load Loss Reduction In System	90 kW	0.475800
Capacity Cost		1.525000
Annual Save		8.035000

Table 2 Optimal solution of general system

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