THE STATE SPACE MYSTERY IN MULTIPLE-VALUED LOGIC CIRCUIT WITH LOAD PLANE – PART I

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SUMMARY

In the article presented we deal with new knowledge of the behaviour of dynamical systems both from theoretical and practical point of view.

Switching sequential circuits are an indispensable part of many modern electronic devices, such as memory cells, flip-flop sensors and many others. Since the invention of flip-flop switching circuits, the study of their dynamic behaviour has played an ever-increasing role. The dynamic properties of sequential circuits can be investigated by means of switching between the system's attractor. In this paper the boundary surfaces are discussed that play a crucial role in the process of switching.

By "analysis of the multivalued logic (MVL) circuit" we understand graphical representation of boundary surfaces that divide the basins of attraction. Each region of attraction contains one stable equilibrium, i.e. a stable singularity or a stable limit cycle. At existence of stable limit cycle are boundary surfaces very complicated and the control of such MVL circuit would most probably be problematic. Therefore we expected that when the stable limit cycles are absent the shape of the boundary surfaces will be simple and therethrough investigated structure would be more simply controlled. The simulation of the MVL structure shown that no always is morphology of the boundary surfaces simple when the stable limit cycles are absent. The knowledge about of morphology of the boundary surfaces corresponding to stable attractors makes it possible to design reliable methods of control of MVL structures.

Keywords: *state space, boundary surface, singularities, piecewise-linear approximation, eigenvalue.*

1. INDRODUCTION

The objective in this paper will be an analysis of a MVL consisting of two resonant tunnel diodes (RTD) connected in series. One of the RTDs is an element and the other represents a load. Since the load exhibits negative dynamic resistance, a natural question arose as to the behavior of this object described by equations (2) later in the text. Concept of the research about MVL circuit was first internally published in [10]. The first simulation results shown extreme distinguished from simulations refer to so-called "classic bistability" in [9], [19].

The very first simulations of the system's behavior suggested that the title STATE SPACE MYSTERY in [3] is not exaggerated at all. Since the RTDs are, according to [21], [1], [7], able to work at gigahertz frequencies, an analysis of these objects has considerable practical significance also. The advantages of MVL from the viewpoint of their transfer to higher orders are described in work [7].

By "analysis of the MVL circuit" we understand graphical representation of boundary surfaces (BS) that divide the basins of attraction. Each region of attraction contains one stable equilibrium, i.e. a stable singularity or a stable limit cycle. The algorithm for the calculation of the BS was first published in [12]. The significance of BS was also described in paper [6], which was included in the book [2], since it deals with boundary surfaces in the context of Chua's circuit and a control pulse.

2. THE CIRCUIT OF THE MULTIVALUED MEMORY

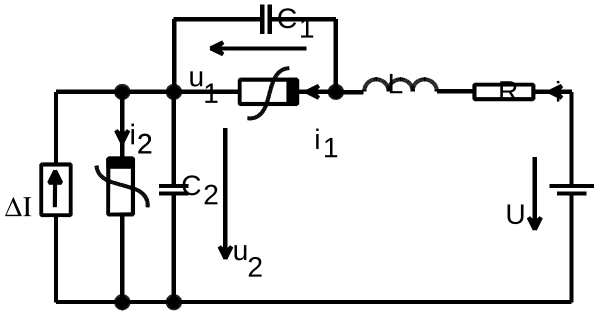

Fig.1 Model of the memory cell

The above-mentioned MVL circuit is shown in *Fig.1* where the symbols of nonlinear elements correspond to resonant tunnel diodes. Let $i_2(u_2)$ and $i_1(u_1)$ represent the *V–I* characteristics of the element and the „negative" load respectively. Both characteristics are piecewise-linear (PWL) characteristics. General algebraic form of the PWL characteristics was first derived in [11]. Characteristics are defined by the expression:

$$f(u) = \frac{1}{2}(g_0 + g_3)u + \frac{1}{2}\big[(g_1 - g_0)\ |u - U_1| + (g_2 - g_1)\ |u - U_2| + (g_3 - g_2)\ |u - U_3|\big] - \frac{1}{2}\big[(g_1 - g_0)U_1 + (g_2 - g_1)U_2 + (g_3 - g_2)U_3\big] \quad (1)$$

Resistance R can represent the resistivity of battery. Inductance L may be parasitic inductance, and C_1, C_2 are parasitic capacitances of the RTD's. The symbol ΔI in *Fig.1* denotes a rectangular current control pulse. After defining all the parameters of the circuit in *Fig.1* we can write

$$\begin{aligned} L\left(\frac{di}{dt}\right) &= U - Ri - (u_1 + u_2) \equiv Q_1 \\ C_1\left(\frac{du_1}{dt}\right) &= i - f_1(u_1) \equiv Q_2 \\ C_2\left(\frac{du_2}{dt}\right) &= i - f_2(u_2) + I \equiv Q_3 \end{aligned} \quad (2)$$

The *V-I* characteristics $f_k(u_k)$ for the active $(k = 2)$ and load device $(k = 1)$ represent surfaces in the state space R^3 and their traces are depicted in the planes $u_1 = 0$, and $u_2 = 0$ in *Fig.2*. The series resistance R may be depicted in the state space through the equality $Q_1 = 0$ which corresponds to a plane. Its traces: α, β, γ are shown in all projection planes: $i = 0,\ u_1 = 0,\ u_2 = 0$ respectively.

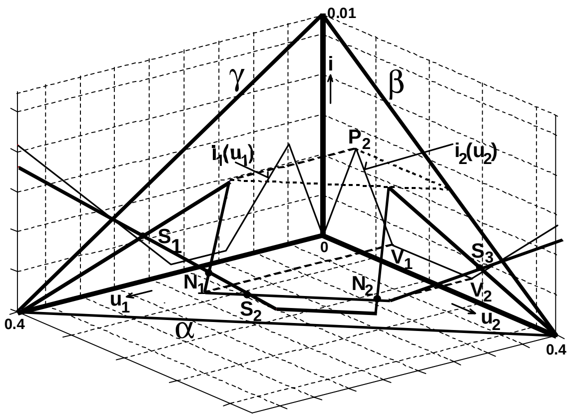

Fig. 2 Traces α, β, γ of the load plane correspond to the equality $Q_1 = 0$ in Eq.2, for resistance $R = 40\Omega$. Parameters of the characteristics are introduced in commentary of *Tab.1*. Singularities S_1, S_2, S_3, N_1, N_2 lie in load plane $Q_1 = 0$.

The determination of the singularities in the Monge projection and their corresponding projections is shown in *Fig.2*. The figure gives us an idea about how dramatically the value R may affects the number of singularities. In the following we will consider only the case $R = 0$ in which case the load plane is perpendicular to the projection plane $i = 0$. The traces of *V-I* characteristics and singularities projected onto the plane $u_1 = 0$ are shown in *Fig.3*. Here it is clear that the change of R to zero value corresponds to five singularities. The geometric determination of the singularities in *Fig.2* was first published in [15].

3. ANALYSIS OF THE CIRCUIT

Dynamic properties of the circuit are most influenced by the character of the singularities and especially saddle points N_1, N_2. Their nature is given by the eigenvalues of the Jacobi matrix. For the system (2) the Jacobi matrix has the form.

$$\mathbf{A} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} & -\frac{1}{L} \\ \frac{1}{C_1} & -\left(\frac{{}^1g_i}{C_1}\right) & 0 \\ \frac{1}{C_2} & 0 & -\left(\frac{{}^2g_i}{C_2}\right) \end{bmatrix} \quad (3)$$

where ${}^k g_i$ correspond to conductances *V-I* characteristics in particular singularity (i = 0, 1, 2, 3 see text for *Tab. 1*).

Eigenvalues are defined through

$$\det\ |\mathbf{A} - \lambda \mathbf{1}| = 0 \quad (4)$$

where λ are the eigenvalues of the matrix **A** and **1** is the unit matrix.

In the case of positive load all the eigenvalues of the saddle points had the property that all but one of them had negative real parts regardless order of the system. This was also corroborated by the particular cases investigated in [12], [18], [13], [8] and [20]. However, during further investigation was discovered that above mentioned postulate has specific constrains. Relationship of eigenvalues to dynamic system will be introduced more exactly in the next papers.

In *Fig.3* we give cross sections of the boundary surfaces surrounding the regions of attraction for oscillatory and static attractors. The values of conductances, inductance, capacitances, eigenvalues of the Jacobi matrix and eigenvectors α_{ij}, including the coordinates of break points of *I-V* characteristics are given in *Tab. 1*.

To depict the basins of attraction we used the grid technique in which every gray-scale point in *Fig.3* corresponds to a trajectory going to its attractor. The gray-scale/attractor correspondence is clear from the Monge projection in *Fig.3*. It should be noted that in *Fig.3* is depicted the cross-section in the plane i, u_2 of the regions of attraction corresponding to the plane $u_1 = 91mV$ which is the value corresponding to the saddle N_2. Similarly, in the plane u_1, u_2 is the cross-section through the singularity N_2 ($i = 2{,}9mA$). There is also the trace of the tangent plane (EG_2) to the double surfaces separating the attractors S_2, S_3. This trace of the tangent plane is tangent to the cross-section of the boundaries in N_2. This simulation test has been first provided in [17].

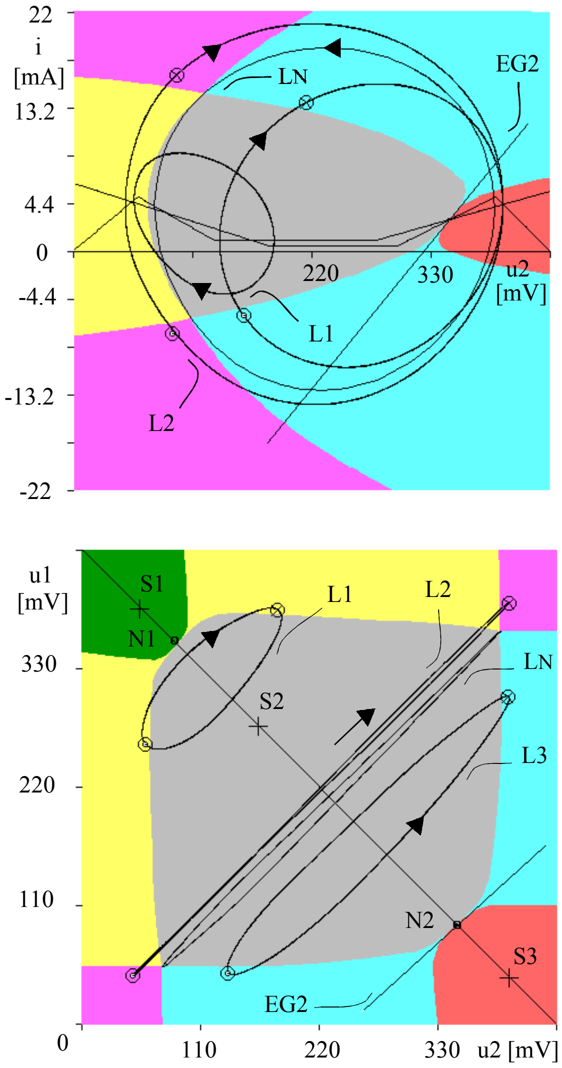

Fig. 3 The Monge's projection of the cross-section (in singularity N_2) of the boundary surfaces and stable limit cycles *(L_1, L_2, L_3)* and unstable limit cycle *(L_N)*.

According to [12] this tangent plane is given by the equation:

$$y_1 = \alpha_{11}\Delta u_1 + \alpha_{12}\Delta u_2 + \alpha_{13}\Delta i = 0 \quad (5)$$

where the eigenvectors α_{ij}, correspond to the dominant eigenvalue of the Jacobi matrix as was first reported in [12]. The values of the eigenvectors for both saddle points are given in *Tab. 1*.

Eq uili br.	co-ordinates			eigenvector		
	u_{i1} [mV]	u_{i2} [mV]	i_i [mA]	α_{i1}	α_{i2}	α_{i3}
S1	385	54	4,5	*	*	*
N1	353	86	3,5	8,2922	-0,8703	1
S2	275	164	1	*	*	*
N2	91	348	2,9	-8,2028	-1,1059	1
S3	43	397	4,3	*	*	*

Eq uili br.	Eigenvalues *x 10* ^{9}		
	λ_{i1}	$Re\{\lambda_{i23}\}$	$Im\{\lambda_{i23}\}$
S1	**-123.213593379**	**-53.693203310**	**185.870855376**
N1	**31.277885807**	**9.461057096**	**178.912952777**
S2	**-32.840127795**	**-15.579936102**	**196.809479969**
N2	**25.828651316**	**8.985674341**	**183.943148711**
S3	**-145.801276658**	**-55.199361671**	**179.155674391**

Tab. 1 The numerical values of the co-ordinates, eigenvalues and eigenvector for equilibria corresponding to memory cell in *Fig.1*. The bias voltage of the memory cell *U=440mV; L=1e–10H,* $C_1=C_2=5e{-}13F$, $R=0\,\Omega$. The parameter values corresponding to active and load device are as follows: ${}^1g_0=0{,}1$; ${}^1g_1=-0{,}05$; ${}^1g_2=0$; ${}^1g_3=0{,}032$; ${}^2g_0=0{,}0833$; ${}^2g_1=-0{,}0571$; ${}^2g_2=0$; ${}^2g_3=0{,}0281[S]$; ${}^1U_1=50$; ${}^1U_2=140$; ${}^1U_3=260$; ${}^2U_1=60$; ${}^2U_2=130$; ${}^2U_3=280$ *[mV]*.

From the viewpoint of morphology the surfaces and the corresponding regions of attraction exhibit unusual properties since in addition to the three stable singularities S_1, S_2, S_3, there are also three stable limit cycles L_1, L_2, L_3 which are not coupled with the nonstable limit cycles as was the case with positive load in the cases described in [6], [9], [13] and [8]. In this case we have only one unstable limit cycle, denoted by L_N in *Fig. 3*. It is located on the border of four regions of attraction corresponding to L_1, L_2, L_3, S_2. On the surface of the bounded region corresponding to singularity S_2 is the limit cycle L_N which was detected in the computer simulation by integration in backward time. Hence L_N is not saddle-type as in the case of positive load in [6], [9], [13] and [8], which means there are no initial conditions whose trajectories are attracted towards L_N as was the case in [18] and [9].

From the practical viewpoint, however, unstable oscillatory phenomena are undesirable. In such a case this analysis is valuable in that it enables to verify the parameter values for which the oscillatory phenomenon is absent. It is most likely that this will occur when the absolute value of negative normed conductance of both RTDs will be less than 1, or the practical value *R>0* will be assummed.

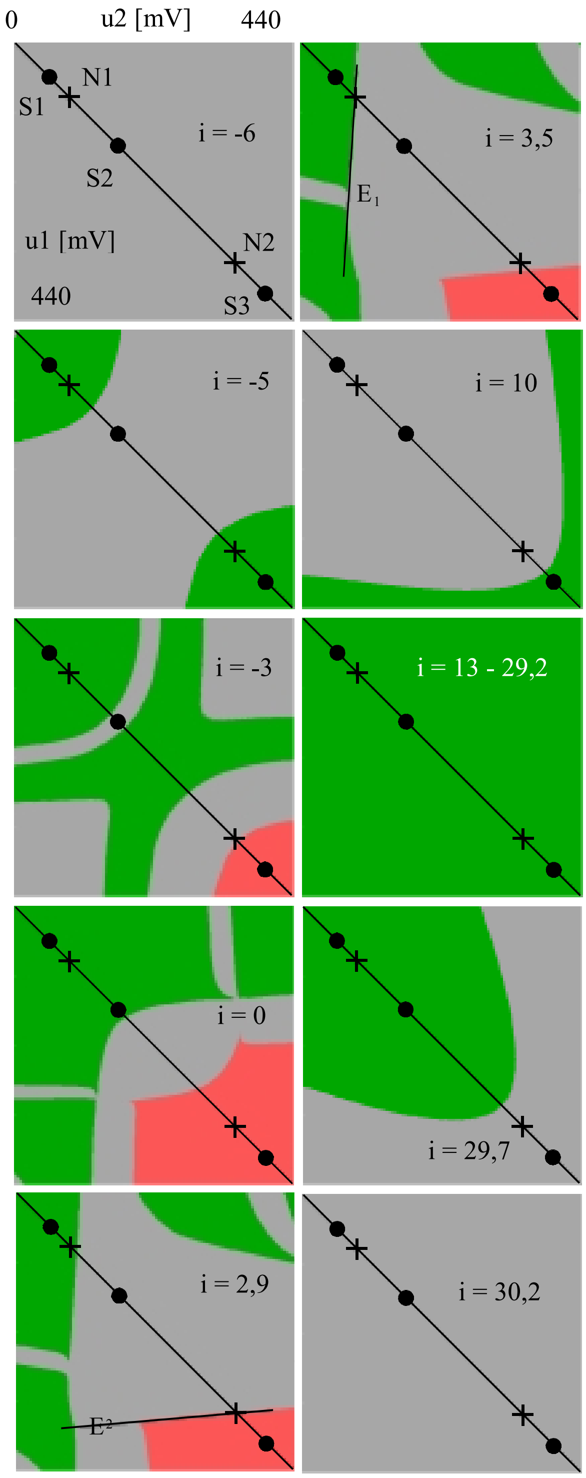

Fig. 4 The cross-sections of the attractors, for corresponding stable states at different current levels and projection onto u_1, u_2-plane. The different gray-scale areas represent the domains of attraction for sinks S_1, S_2, S_3. Depicted are the both traces of tangential planes E_1, E_2 as well. Capacitances were chosen $C_1 = C_2 = 3e-14F$.

4. MORPHOLOGY OF THE BOUNDARY SURFACE WHEN LIMIT CYCLES ARE ABSENT

In *Fig. 4, Fig. 5* and *Fig. 6* we present cross sections of boundary surfaces, when the limit cycles are absent. The capacitances of the circuit in *Fig.1* are the bifurcation parameters. *Fig.4,* and *Fig. 5* are related to the values $C_1 = C_2 = 3e-14F$ and $C_1 = C_2 = 4e-14F$ respectively. The eigenvalues of the Jacobi matrix corresponding to the saddle point N_1 for $C_1 = C_2 = 3e-14F$ are

λ_1 =17.22358266519e+11
λ_2 =-6.27750395530e+11
λ_3 =-2.57941204322e+11

and similarly the eigenvalues for the saddle point N_2 are

λ_1 =14.57963369881e+11
λ_2 =-3.63981684940e+11 -1.85517698577e+11i
λ_3 =-3.63981684940e+11 +1.85517698577e+11i

Interesting in this case is the shape of the region of attractions. The darkest gray area corresponds to region of attraction of the singularity S_1, the lightest gray area corresponds to region of attraction of S_2 and gray area corresponds to region of attraction of S_3. The cross section at current level $i = -6mA$ (in *Fig. 4* it match to note $i = -6$, the same convention is used for all subfigures) is the first one to show the region of attraction only for stable singularity S_2. At the level $i = -5mA$ this region is branching, whereas from $i = -3mA$ to $i = 3{,}5mA$ there are already very complicated structure of boundary surfaces. At higher current levels there is the morphology of boundary surfaces simple again. From current $13-29.2mA$ there is the region of attraction only for stable singularity S_1. The shape of the region of attractions can be clearer, as well, from cross-sections parametrized by u_1 in *Fig.5* (notation of the cross sections at different voltage levels and different gray-scale areas of attraction used in this figure are the same as mentioned for *Fig.4*). The sections, parametrized by u_1, clearly document the morphological complexity of the corresponding regions. Therefore the control of such a ternary would most probably be problematic.

More simple shape of the region of attractions (useful for the control of such a ternary, for example) is depicted in *Fig. 6*. The capacitances of the circuit in *Fig.1* correspond to the regions of attraction in *Fig. 6* was chosen $C_1 = C_2 = 9e-14F$.

Notation of the cross sections at different current levels and different gray-scale areas of attraction used in this figure are the same as mentioned for *Fig.4*.

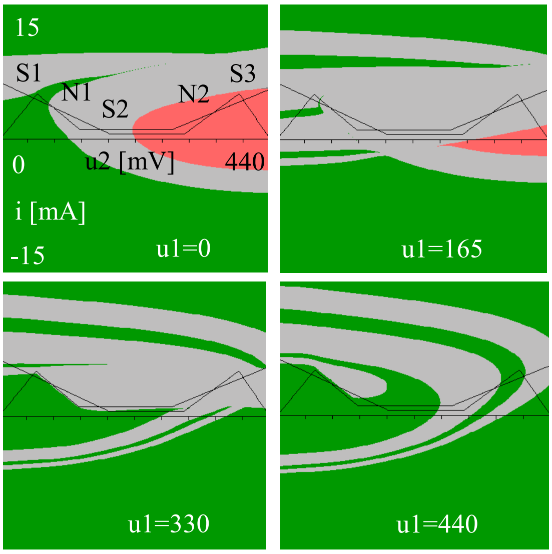

Fig. 5 The cross-section of the attractors, for corresponding stable states at different voltage levels (u_1) and projection onto i, u_2-plane. The different gray-scale areas represent the domains of attraction for sinks S_1, S_2, S_3. Capacitances were chosen $C_1 = C_2 = 4e-14F$.

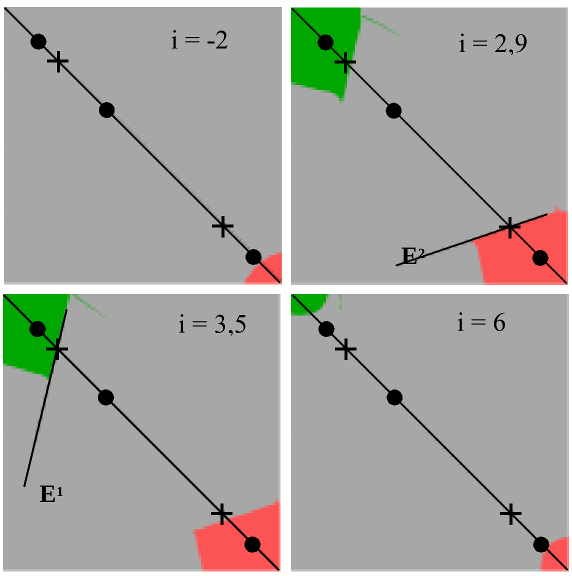

Fig. 6 The cross-section of the attractors, for corresponding stable states at different current levels and projection onto u_1, u_2-plane. The different gray-scale areas represent the domains of attraction for sinks S_1, S_2, S_3. Depicted are the both tangential planes E_1, E_2 as well. Capacitances were chosen $C_1 = C_2 = 9e-14F$.

5. CONCLUSION

From the viewpoint of multiple-valued logic based on RTD diodes the most significant result is one corresponding to the nonzero *R* in *Fig.1*. Those values of *R* that were considered in this contribution could correspond to the resistance of the sources to which the circuit is connected. The fact how dramatically the number of singularities is affected by the value of *R* was first published in [15]. The method of analysis of MVL, demonstrated in this contribution gives the possibility of reliable design of MVL structures.

The morphology of the basins of attraction, corresponding to stable attractors makes it possible to design reliable methods of control of MVL structures, which was first published in [16], [14], and later in [18] and [4].

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BIOGRAPHY

Viktor Špány (Prof., Ing., DrSc.), received his DrSc (PhD) degree from the Slovak University of Technology in Bratislava, Czechoslovakia. After joining the University of Technology in Košice in 1952 his research was devoted to pulse circuits and digital electronics. The result of these activities, published in local and international journals, have been summarized in the book Bipolar Transistor in Pulse Circuits. He directed his further activities toward numerical and graphical solutions of non-linear dynamical systems. Among the most important results were the algorithms for construction and utilization of boundary surfaces in flip-flop circuits and oscillatory systems. Currently he is Professor Emeritus of electrical engineering at the Department of Electronics and Multimedial Communications, Technical University in Košice, Slovakia.

Pavol Galajda was born in 1963 in Košice, Slovak Republic. He received the Ing. (M.Sc.) degree in electrical engineering from the FE TU in Košice and CSc. (Ph.D.) degree in radioelectronics from FEI TU in Košice, in 1986 and 1995, respectively. At present he is an assistant professor at the Department of Electronics and Multimedial Communications, FEI TU in Košice. His research interest is in nonlinear circuits theory and multiple-valued logic.

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