BOWING OF THE MOVING BOUNDARY BETWEEN CIRCULAR DOMAINS

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SUMMARY

The profile of a boundary between circular domains (circular domain wall-CDW) is calculated in the first-order approximation. On the basis of this calculation critical current I_c for collapse of CDW was derived. The value of I_c for stress annealed $Co_{68.2}Fe_{4.3}Si_{12.5}B_{15}$ wire is close to the upper limit of the current interval in which the motion of a single CDW can be measured. The effect of CDW bowing is therefore probably not negligible in these experiments.

Keywords: *magnetic domain wall, amorphous wires, magnetic domain wall mobility, magnetic domain wall bowing, circular magnetization*

1. INTRODUCTION

Velocity of a single boundary between circular domains (circular domain wall - CDW) as a function of a constant current (circular field) flowing through cylindrical sample has been recently measured in stress annealed amorphous $Co_{68.2}Fe_{4.3}Si_{12.5}B_{15}$ wire [1,2]. For a region of currents where the velocity versus current dependence can be considered as linear agreement between experimental CDW mobility and the one obtained using the model of a rigid plane CDW is very good [1]. However, the experiments with different directions of the current flow with respect to the direction of CDW movement [2] indicate that even in this region of currents the effect of the so called "domain drag" is probably superimposed by some other mechanism.
None of the fields (driving circular field due to current and eddy current field) are uniform in the CDW. It raises the question of possible distortion of CDW due to variations of the pressure these fields exert on the CDW. The simplest way how to estimate the degree of CDW distortion consists in finding the wall profile for which variations of the field, in a plane CDW as a zeroth-order approximation, are balanced by variations in curvature [3]. Based on this idea we present calculations of CDW profile and critical field for a collapse of CDW. The degree of CDW distortion for experiments presented in [1,2] is also briefly discussed.

2. BASIC EQUATIONS

Consider a cylindrical sample with two circular domains separated by a single CDW as shown in Fig.1. On condition that the radius of the axially magnetized region in the middle of the cylinder is much less then the radius of the cylinder R we can neglect this region in our model. The driving field $H_{\varphi a}$, created by current I flowing along the cylinder, sets the CDW into motion. If the velocity of CDW v_w is constant the equation of motion is [3]

$$\frac{\varepsilon}{2\mu_0 M_s} C + H_\varphi = 0\,, \tag{1}$$

where C is the wall curvature, ε is the wall energy per unit area, M_s is the saturation magnetization, $H_\varphi = H_{\varphi a} + H_e$ and H_e is the eddy current field.
Considering cylindrical symmetry the equation of the CDW is given by a function $z = z_w(r)$. Let the coordinate system be connected with the moving CDW in such a way that $z_w(0) = 0$. The wall curvature C is a sum of principal curvatures of the CDW:

$$C = \frac{1}{r}\frac{d}{dr}\left\{ \frac{r\dfrac{dz_w}{dr}}{\left[1 + \left(\dfrac{dz_w}{dr}\right)^2\right]^{\frac{1}{2}}} \right\} = \frac{1}{Rx}\frac{d}{dx}\left\{ \frac{x\dfrac{dZ_w}{dx}}{\left[1 + \left(\dfrac{dZ_w}{dx}\right)^2\right]^{\frac{1}{2}}} \right\}, \tag{2}$$

where $x = \dfrac{r}{R}$, $Z = \dfrac{z}{R}$ and $Z_w = \dfrac{z_w}{R}$.

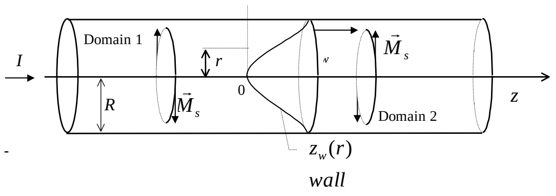

Fig.1 The model of a cylindrical sample with a single boundary between two circular domains

3. CALCULATION OF THE MAGNETIC FIELD IN THE ZEROTH ORDER APPROXIMATION

To calculate the field in a planar CDW we use expressions for z-component of current density [1]:

$$j_{1z}(x,Z)=\sum_{n=1}^{\infty}C_nJ_0(\mu_nx)e^{\mu_nZ}+\frac{I}{\pi R^2}$$
$$j_{2z}(x,Z)=\sum_{n=1}^{\infty}C_nJ_0(\mu_nx)e^{-\mu_nZ}+\frac{I}{\pi R^2}, \qquad (3)$$

where $C_n=\frac{\mu_0M_sv_w\pi}{\mu_n\rho}\frac{\mathrm{H}_1(\mu_n)}{J_0(\mu_n)}$, H_1 is Struve function [4]; subscript 1, 2 refers to the domain 1 and 2, respectively; μ_n are zeros of the Bessel function of first order J_1, and J_0 is Bessel function of zero order.

Eqs. 3 were derived for the CDW located at $z=0$. Using Amper‘s law the zeroth-order field $H_\varphi^{(0)}$ in the wall is:

$$H_\varphi^{(0)}=\frac{1}{r}\int_0^r j_z(r,0)rdr=\frac{R}{x}\int_0^x j_z(x,0)xdx \qquad (4)$$

then

$$H_\varphi^{(0)}=R\sum_{n=1}^{\infty}\frac{C_n}{\mu_n}J_1(\mu_nx)+\frac{I}{2\pi R}x\,. \qquad (5)$$

If CDW moves at constant velocity the net force acting on it has to be equal to zero. This condition is fulfilled when

$$\iint_S H_\varphi^{(0)}dS=\int_0^R H_\varphi^{(0)}2\pi rdr=0\,, \qquad (6)$$

where integration is over the whole CDW.

By substituting $H_\varphi^{(0)}$ and carrying out indicated calculation we obtain for the wall velocity:

$$v_w=\frac{I}{3\pi^3\Omega}\frac{\rho}{\mu_0M_sR^2}\,, \qquad (7)$$

where $\Omega=\sum_{n=1}^{\infty}\frac{\mathrm{H}_1^2(\mu_n)}{\mu_n^3}$.

The expression for the wall velocity obtained in this way is the same as the one obtained by comparing eddy current power loss with power supplied by the applied circular field acting on the CDW [1].

Now the velocity can be substituted into the expression for C_n and using (5) the field in a planar CDW moving at constant velocity driven by current I is

$$H_\varphi^{(0)}=h(x)\frac{I}{2\pi R} \qquad (8)$$

where

$$h(x)=\frac{2}{3\pi\Omega}\sum_{n=1}^{\infty}\left[\frac{\mathrm{H}_1(\mu_n)}{\mu_n^2J_0(\mu_n)}\right]J_1(\mu_nx)+x\,.$$

The function $h(x)$ characterizes the variation of the field in CDW. Its values were calculated for the first twenty members of the series and the result is shown in Fig.2.

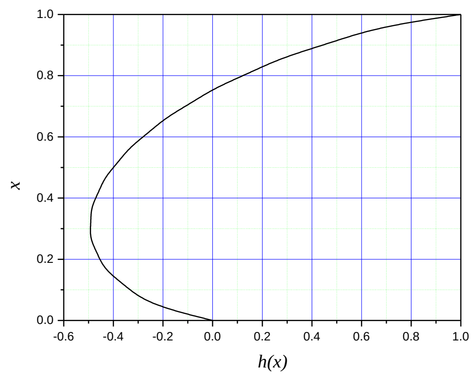

Fig.2 The field variation in the plane CDW

4. CALCULATION OF THE WALL PROFILE IN THE FIRST ORDER APPROXIMATION

We consider the wall moving at constant velocity. In the first-order approximation we assume that at each point of the wall the force due to the field given by (8) is compensated by the force due to bowing:

$$2\mu_0 M_s H_\varphi^{(0)} =$$

$$= -\varepsilon \frac{1}{Rx}\frac{d}{dx}\left\{ \frac{x\frac{dZ_w^{(1)}}{dx}}{\left[1+\left(\frac{dZ_w^{(1)}}{dx}\right)^2\right]^{\frac{1}{2}}} \right\} \quad (9)$$

Using (8) and after first integration we have

$$\alpha f(x) = \frac{\frac{dZ_w^{(1)}}{dx}}{\left[1+\left(\frac{dZ_w^{(1)}}{dx}\right)^2\right]^{\frac{1}{2}}}, \quad (10)$$

where

$$f(x) =$$
$$= \frac{1}{\Omega}\sum_{n=1}^{\infty}\frac{\mathrm{H}_1(\mu_n)}{\mu_n^3 J_0(\mu_n)}\left[J_0(\mu_n x)\mathrm{H}_1(\mu_n x) - \right. \quad (11)$$
$$\left. - J_1(\mu_n x)\mathrm{H}_0(\mu_n x)\right] - x^2$$

and $\alpha = \frac{\mu_0 M_s I}{3\pi\varepsilon}$.

The integration constant is equal to zero since the expression in braces $\{...\}$ in Eq. 9 equals to zero for $x = 0$.

Taking the first twenty members of the series in (11) the function $f(x)$ is plotted in Fig.3. This function has a maximum of about 0.382. The left hand side of Eq.10 cannot be greater then unity. Putting the left hand side equal to unity and taking maximum value of $f(x)$ we can find critical current

$$I_c = \frac{3\pi\varepsilon}{0.382\times\mu_0 M_s}. \quad (12)$$

For currents higher then I_c the curvature can no longer compensate for variation in the circular field H_φ and the wall may be expected to collapse.

By rearrangement of Eq.10 we obtain:

$$\frac{dZ_w^{(1)}}{dx} = \frac{\alpha f(x)}{\left[1-(\alpha f(x))^2\right]^{\frac{1}{2}}} \quad (13)$$

The CDW profiles obtained by numerical integration of Eq. 13 are plotted in Fig.4.

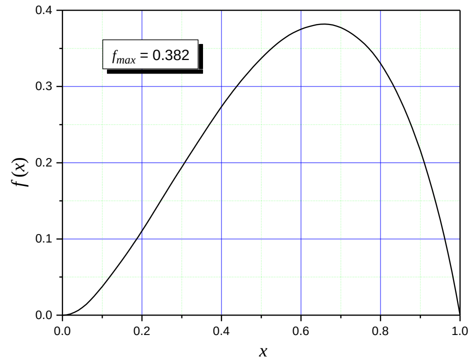

Fig.3 Function $f(x)$

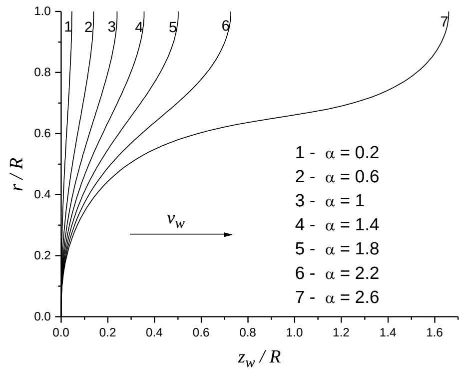

Fig.4 CDW profiles calculated for different values of parameter α

5. COMPARISON WITH EXPERIMENT

The experiments with single CDW carried out on the stress annealed $Co_{68.2}Fe_{4.3}Si_{12.5}B_{15}$ wire were reported in [1,2]. For currents up to about 3mA the velocity of the CDW versus current is well described by linear dependence. For currents higher than about 3.5mA the rate of magnetization reversal increases so rapidly that it can no longer be considered as a result of the single wall movement. This current value was found to be the upper limit for single CDW experiments. It is interesting that critical field for creation of reverse circular domains is slightly higher (about 4.2 mA).

The degree of CDW distortion for a given current can be characterized by the value of $Z_w^{(1)}(1)$. For experiments presented in [1,2] we can make the following estimate. Supposing Landau-Lifshitz structure of the wall the energy per unit area of the wall is [5]:

$$\varepsilon = 4\sqrt{KA}, \quad (14)$$

where K, A are anisotropy and exchange constants, respectively.

The value of anisotropy constant K derived from the axial magnetization curve is about $257\,\mathrm{Jm}^{-3}$. Taking exchange constant $A = 4 \times 10^{-12}\ \mathrm{Jm}^{-1}$ and $\mu_0 M_s = 0.735\,\mathrm{T}$ [6] we obtain $I_c \approx 4.3\,\mathrm{mA}$ and $\alpha \approx 608 \times I$. The dependence $Z_w^{(1)}(1)$ versus current is plotted in Fig.5.

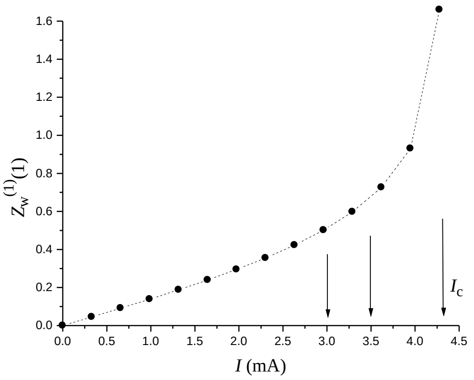

Fig.5 The degree of CDW distortion as a function of a current calculated for parameters of stress annealed $Co_{68.2}Fe_{4.3}Si_{12.5}B_{15}$ amorphous wire

We can see that $Z_w^{(1)}(1)$ is about 0.5 for current of 3 mA so we can conclude that the distortion of CDW is probably not negligible for experiments on stress annealed CoFeSiB wire. The value of I_c is not much higher than the upper limit (3.5mA) for the single CDW experiment. Having in mind that the value of I_c is rough estimate we can not exclude the possibility that it is the collapse of CDW which does not allow to perform the single CDW experiments for currents higher than 3.5mA.

6. CONCLUSIONS

The profile of a boundary between circular domains was calculated in the first-order approximation. On the basis of this calculation critical current I_c for collapse of CDW was derived. The value of I_c for stress annealed $Co_{68.2}Fe_{4.3}Si_{12.5}B_{15}$ wire is close to the upper limit of the current interval in which the motion of a single CDW can be observed. It is possible that the collapse of CDW occurs for currents less than critical current needed for nucleation of reverse circular domains.

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