MODELLING OF RESISTANCE AND INDUCTANCE OF NONFERROMAGNETIC CONDUCTOR WITH RESPECTING ITS DIRECT HEATING

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SUMMARY

The paper deals with simulation of electromagnetic and thermal phenomena in a nonferromagnetic cylindrical conductor of circular cross-section fed from a source of general time-dependent voltage. The task is formulated as a hard-coupled electromagnetic-thermal problem that is solved in common with the computation of the corresponding time evolution of the current. Its mathematical model consists of two partial differential equations describing both the fields and an ordinary differential equation for the time dependence of the current. These equations supplemented with correct initial and boundary conditions are solved by the FDM using an appropriate explicit-implicit approximation. The methodology is illustrated on a task whose results are discussed.

Keywords: *effective resistance, effective internal inductance, skin effect, electromagnetic field, temperature field, finite differences*

1. INTRODUCTION

Analysis of the voltage and current surge phenomena in electrical systems as well as in windings of the rotating machines and transformers usually does not take into account variances of the resistance and internal inductance due to the skin effect. The reason consists in a wide-spread opinion that increase of the resistance brings about damping of the surge effects and its neglecting leads to higher values of the investigated quantities. And when the devices are designed correspondingly, their safety is higher.

In case of pulses with steep front, however, the effective resistance may be even by an order higher than the DC resistance. Neglecting the skin effect can lead, therefore, to unacceptably distorted ideas about the voltage and current phenomena in the device. The situation (particularly in accidental states) is also affected by temperature rise of the current-carrying parts, which brings about a supplementary increase of their resistances.

Thus, complete analysis of the task represents a coupled electromagnetic-thermal problem and the paper offers a methodology how to cope with it. We consider a surge skin effect in a conductor of circular cross section placed in medium from which the produced heat is transferred by convection (which is usual, for example, at various kinds of grounders, or overhead and cable lines in power distribution systems).

2. FORMULATION OF THE PROBLEM

A cylindrical conductor of circular cross-section (radius r_0, length l_0, see Fig. 1) is supplied from a source of pulse voltage $u(t)$. Its material is nonferromagnetic ($\mu = \mu_0$) and temperature dependencies of its parameters (electrical conductivity $\gamma(T)$, thermal conductivity $\lambda(T)$, specific mass $\rho(T)$ and specific heat $c(T)$) are supposed to be known.

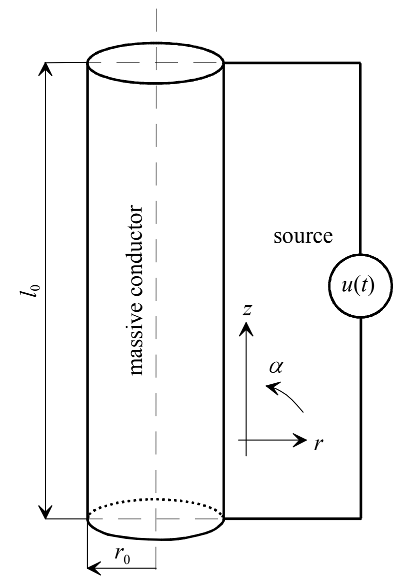

Fig. 1 The investigated disposition

The aim is to find the time evolution of the effective resistance $R_{\text{eff}}(t)$ and internal inductance $L_{\text{eff}}(t)$ that are influenced by

- the shape of the current pulse,
- corresponding time-dependent distribution of the current density along the radius,
- consequent temperature rise due to the Joule losses in the conductor.

As the solution must respect mutual influence of all above quantities, the task will be handled as a hard-coupled problem. This means that all required quantities will be calculated simultaneously.

3. MATHEMATICAL MODEL OF THE PROBLEM

The basic mathematical model of the problem consists of two partial differential equations describing the

- nonstationary electromagnetic field expressed by distribution of the vector potential $\boldsymbol{A}(\boldsymbol{r},t)$,
- nonstationary $T(\boldsymbol{r},t)$ temperature field produced by the Joule losses w_{J},

and one ordinary differential equation for current $i(t)$ in the massive conductor. These equations are supplemented with relevant initial and boundary conditions.

The nonstationary electromagnetic field within the conductor is described by equation (see, for example, [1])

$$\frac{1}{\mu_0}\mathrm{rot\,rot}\,\boldsymbol{A}+\gamma\frac{\partial \boldsymbol{A}}{\partial t}=\boldsymbol{J}_0, \qquad (1)$$

where $\boldsymbol{J}_0$ denotes the vector of the uniform current density in the conductor corresponding to external current $i(t)$ from the source. In this case $\boldsymbol{J}_0$ has only one nonzero component $J_{0z}(t)$ and $\boldsymbol{A}$ also one nonzero component $A_z(r,t)$. Now the equation (1) within the conductor $\Omega \equiv \{0 \le r \le r_0, 0 \le t\}$ may be transcripted as

$$\frac{\partial^2 A_z}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial A_z}{\partial r}-\mu_0\gamma\frac{\partial A_z}{\partial t}=-\mu_0 J_{0z}. \qquad (2)$$

Unambiguousness of its solution is assured by the following boundary and initial conditions:

- $$\frac{\partial A_z}{\partial r}=0 \ \text{ for } \ r=0 \text{ and } t\ge 0 \qquad (3)$$
  (rotation symmetry),
- $$\gamma\int_0^{r_0}\frac{\partial A_z}{\partial t}\cdot 2\pi r\,\mathrm{d}r=0 \ \text{ for } \ r=r_0,\ t\ge 0 \qquad (4)$$
  (an indirect condition expressing the fact that in case of a voltage source the eddy currents close within the massive conductor within any external effects),
- $$A_z(r,t)=0 \ \text{ for } \ 0\le r\le r_0,\ t=0 \qquad (5)$$
  (at the beginning no magnetic field is supposed to affect the arrangement).

The knowledge of distribution of the vector potential $A_z(r,t)$ then provides:

- distribution of the eddy current density $J_{\mathrm{eddy}}(r,t)$ within the conductor
  $$J_{\mathrm{eddy}}=\gamma\cdot\frac{\partial A_z}{\partial t}, \qquad (6)$$
- specific Joule losses $w_{\mathrm{J}}(r,t)$ within the conductor
  $$w_{\mathrm{J}}=\frac{1}{\gamma}\cdot\left(J_0+J_{\mathrm{eddy}}\right)^2, \qquad (7)$$
- total Joule losses $W_{\mathrm{J}}(t)$ in the conductor of length l_0
  $$W_{\mathrm{J}}=l_0\cdot\int_0^{r_0} w_{\mathrm{J}}\cdot 2\pi r\,\mathrm{d}r, \qquad (8)$$
- effective resistance $R_{\mathrm{eff}}(t)$ of the conductor of length l_0
  $$R_{\mathrm{eff}}=\frac{W_{\mathrm{J}}}{i_0^2}. \qquad (9)$$

In a similar manner we can determine the effective inductance $L_{\mathrm{eff}}(t)$ of the conductor.

- The magnetic flux density within the conductor has only the tangential component $B_\alpha(r,t)$ that may be expressed as
  $$B_\alpha=-\frac{\partial A_z}{\partial r}. \qquad (10)$$
- The corresponding volume energy $w_{\mathrm{m}}(r,t)$ of the magnetic field is
  $$w_{\mathrm{m}}=\frac{1}{\mu_0}\cdot B_\alpha^2 \qquad (11)$$
- and the total energy $W_{\mathrm{m}}(t)$ in the conductor of length l_0
  $$W_{\mathrm{m}}=l_0\cdot\int_0^{r_0} w_{\mathrm{m}}\cdot 2\pi r\,\mathrm{d}r. \qquad (12)$$
- Hence, the effective inductance $L_{\mathrm{eff}}(t)$ is
  $$L_{\mathrm{eff}}=\frac{2W_{\mathrm{m}}}{i_0^2}. \qquad (13)$$

The nonstationary temperature field is described by equation (see, for example [2])

$$\mathrm{div}\left(\lambda\,\mathrm{grad}\,T\right)=\rho c\cdot\frac{\partial T}{\partial t}-w_{\mathrm{J}}, \qquad (14)$$

where temperature $T=T(r,t)$ and specific Joule losses w_{J} are given by (7). In our case this equation within the conductor may also be transcripted as

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial T}{\partial r}=\frac{\rho c}{\lambda}\cdot\frac{\partial T}{\partial t}-\frac{w_{\mathrm{J}}}{\lambda}. \qquad (15)$$

Unambiguousness of its solution is assured by the following boundary and initial conditions:

- $$\frac{\partial T}{\partial r}=0 \ \text{ for } \ r=0,\ t\ge 0, \qquad (16)$$
  (rotation symmetry),
- $$-\lambda\cdot\frac{\partial T}{\partial r}=\alpha_{\mathrm{c}}\cdot\left(T-T_0\right) \ \text{ for } \ r=r_0,\ t\ge 0 \qquad (17)$$
  (convective transfer of heat from the conductor surface into ambient air of temperature T_0, α_{c} being the corresponding coefficient),
- $$T(r,t)=T_{\mathrm{s}} \ \text{ for } \ 0\le r\le r_0,\ t=0 \qquad (18)$$
  (at the beginning of the process the conductor has initial temperature T_{s}).

The circuit equation may be written in the form

$$R_{\mathrm{eff}}(t)\cdot i_0(t)+i_0(t)\cdot\frac{\mathrm{d}L_{\mathrm{eff}}(t)}{\mathrm{d}t}+L_{\mathrm{eff}}(t)\cdot\frac{\mathrm{d}i_0(t)}{\mathrm{d}t}=u(t). \qquad (19)$$

with the following initial conditions:

- $$i_0=0 \ \text{ for } \ t=0, \qquad (20)$$
- $$R_{\mathrm{eff}}(0)=R_0=\frac{l_0}{\gamma\pi r_0^2}, \qquad (21)$$
- $$L_{\mathrm{eff}}(0)=L_0=\frac{\mu_0 l_0}{8\pi}, \ \text{ see, for example, [3].} \qquad (22)$$

All the presented equations represent an interconnected system describing a coupled electromagnetic-thermal problem in combined field-circuit interpretation.

4. ALGORITHM OF THE NUMERICAL SOLUTION

The definition area $\Omega \equiv \{0 \le r \le r_0,\ t \ge 0\}$ is first discretised in the following manner:

- in direction r by a set of N points r_i, $i = 1, \ldots, N$ ($r_1 = 0$, $r_N = r_0$) with step Δr,
- in direction t by a system of time levels t_1, t_2, …, t_l, t_{l+1}, … with step Δt.

The algorithm itself then consists of these steps:

a) Computation of values $A_{zi,l+1}$ from values $A_{zi,l}$, where $i = 2, \ldots, N-1$, by means of the explicit difference approximation of (2) (see, for instance, [4]), with respecting the dependence $\gamma_{i,l} = \gamma\left(T_{i,l}\right)$, see eq. (23).

b) Computation of values $A_{z1,l+1}$ and $A_{zN,l+1}$ by means of implicit approximations of eq. (3) or (4).

c) Determination of values $T_{i,l+1}$ from values $T_{i,l}$, where $i = 2, \ldots, N-1$, by means of explicit difference approximation of (15) with respecting dependencies $\lambda_{i,l} = \lambda\left(T_{i,l}\right)$, $\rho c_{i,l} = \rho c\left(T_{i,l}\right)$ (see Fig. 3) and using relation (7) for calculation of $w_{\mathrm{J}i,l}$.

d) Calculation of $T_{1,l+1}$ and $T_{N,l+1}$ by means of implicit approximations of eq. (16) and (17).

e) Computation of $R_{\mathrm{eff}\ l+1}$ by means of numerical approximation of relations (6) – (9) and knowledge of values $A_{zi,l}$ and $A_{zi,l+1}$, $i = 1, \ldots, N$.

f) Computation of $L_{\mathrm{eff}\ l+1}$ by means of numerical approximation of relations (10) – (13) and knowledge of values $A_{zi,l}$ and $A_{zi,l+1}$, $i = 1, \ldots, N$.

g) Computation of $i_{0,l+1}$ from the explicit difference approximation of (19).

The computation is repeated until a prescribed time T_{stop} is reached.

5. ILLUSTRATIVE EXAMPLE

The task is to determine the time evolution of the effective resistance $R_{\mathrm{eff}}(t)$ and inductance $L_{\mathrm{eff}}(t)$ of a direct massive copper conductor of length $l_0 = 100$ m and radius $r_0 = 0.004$ m that is supplied by a voltage pulse depicted in Fig. 2. Calculations should respect the influence of temperature rise of the conductor. Further necessary data follow:

- starting temperature of the conductor $T_s = 30$°C,
- temperature of ambient air $T_0 = 20$°C,
- ambient air does not move, so that the corresponding value of the heat transfer coefficient (see, for example [6]) $\alpha_c = 20$ W/m^{2}K.
- dependence $\gamma = \gamma(T)$ for copper is given [7] as

$$\gamma = \frac{1}{\rho_{20} \cdot \left(1 + \alpha_{\mathrm{p}}\left(T - 20\right)\right)}, \tag{23}$$

where $\rho_{20} = 1.673 \cdot 10^{-8}$ Ωm,

$\alpha_{\mathrm{p}} = 0.0043$ K^{-1},

- the direct-current resistance R_0 of the conductor (21) is $3.362 \cdot 10^{-2}$ Ω,
- the direct-current inductance L_0 of the conductor (22) is $5 \cdot 10^{-6}$ H,
- dependencies $\lambda(T)$ and $\rho c(T)$ for copper [5] are depicted in Fig. 3.

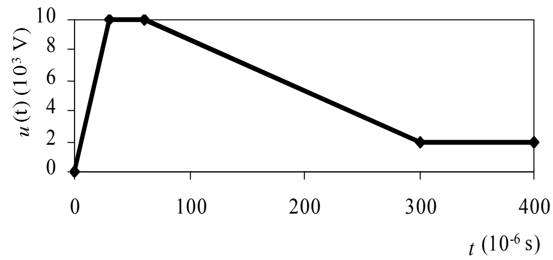

Fig. 2 The shape of the voltage pulse $u(t)$

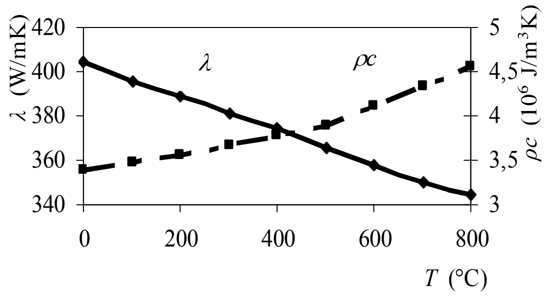

Fig. 3 Temperature dependencies of $\lambda(T)$ and $\rho c(T)$ for copper

All computations have been carried out by a single-purpose user program RL_EF developed and written by the authors. Several results are presented in the following figures.

Fig. 4 contains the time evolution of the total current $i_0(t)$ in the conductor produced by the voltage pulse in Fig. 2. The current obviously lags behind the voltage $u(t)$. This is caused by relation of quantities $u(t)$, $i_0(t)$ and $R_{\mathrm{eff}}(t)$ (see Figs. 3, 4 and 7) occurring in (19) that admit:

$$u(t) > R_{\mathrm{eff}} \cdot i_0(t) \Rightarrow \frac{\mathrm{d}i_0}{\mathrm{d}t} > 0$$

even when $u(t)$ decreases (the influence of term $\mathrm{d}L_{eff}/\mathrm{d}t$ is practically negligible, as can be seen from Fig. 8).

Fig. 5 contains the time evolution of the total Joule losses $W_{\mathrm{J}}(t)$ produced by the distribution of the total current density. These losses are considerably high and, obviously, influence of the consequent temperature rise has to be taken into account. On the other hand, the losses are produced only for a very short time, so that the total heat is relatively small and the temperature rise can be expected not to exceed the admissible value.

Fig. 6 containing the distribution $T(r,t)$ within the conductor fully confirms the conclusions following from the discussion to Fig. 5. Well visible is the influence of cooling by ambient air; temperatures along the surface of the conductor (r = 0.004 m) reach in time t = 400 μs practically two times smaller values than along its axis (r = 0 m)

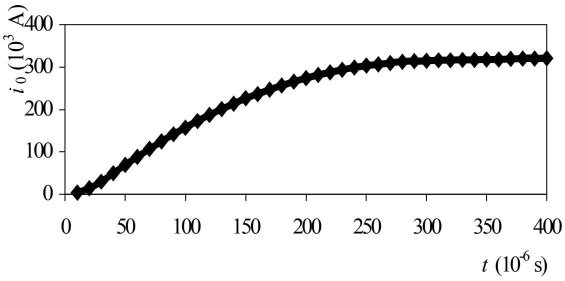

Fig. 4 Time evolution of total current $i_0(t)$

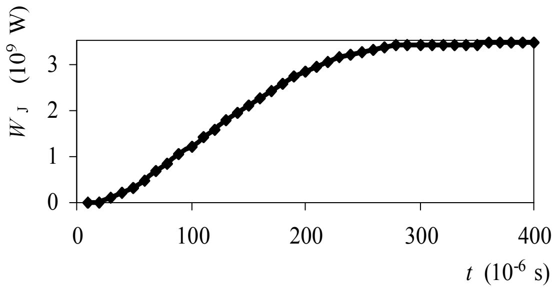

Fig. 5 Time evolution of the total Joule losses $W_J(t)$

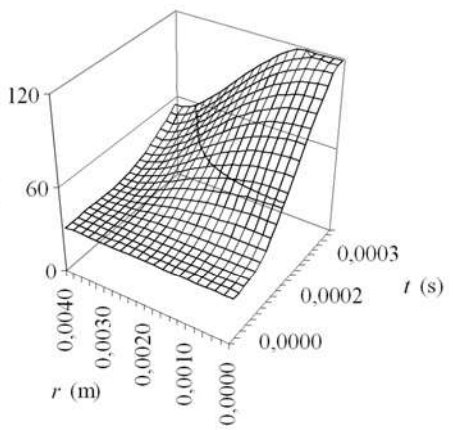

Fig. 6 Distribution of the temperature $T(r,t)$ within the conductor

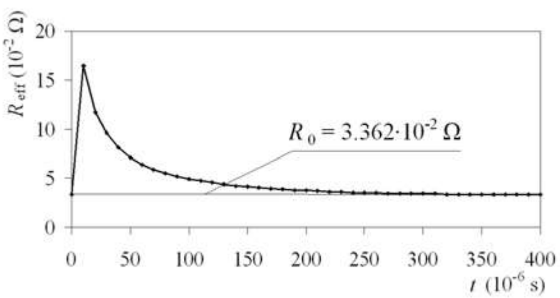

Fig. 7 Time evolution of $R_{eff}(t)$

The final results are presented in Figs. 7 and 8 containing the time evolution of the effective resistance $R_{eff}(t)$ and effective inductance $L_{eff}(t)$. It is obvious that:

- the effective resistance $R_{eff}(t)$ is higher than the direct-current resistance R_0 (see eq. (21)),
- the effective inductance $L_{eff}(t)$ is lower than the direct-current inductance L_0 (see eq. (22)).

Both functions $R_{eff}(t)$ and $L_{eff}(t)$, however, asymptotically approach the values R_0 and L_0, respectively, which is in accordance with theory.

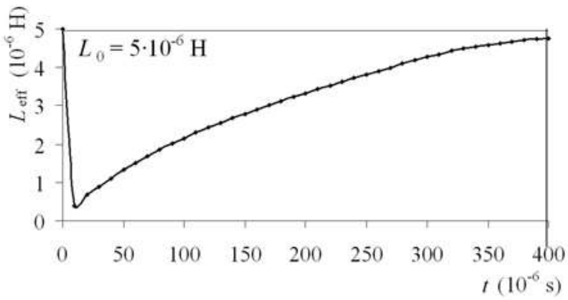

Fig. 8 Time evolution of $L_{eff}(t)$

6. CONCLUSION

The presented methodology allows solving this hard-coupled electromagnetic-thermal problem with good accuracy and reasonable time of computation. Next work in the field will be aimed at extending the algorithm by conductors of more general cross-sections, mutual influence of several near conductors (proximity effect), influence of ferromagnetic material etc.

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BIOGRAPHY

Prof. Ivo Doležel (1949) obtained his Eng. degree from the Faculty of Electrical Engineering (Czech Technical University in Prague) in 1973 and after 28 years in the Institute of Electrical Engineering of the Academy of Sciences of the Czech Republic he returned back to the Czech Technical University. He is aimed mainly at electromagnetic fields and coupled problems in heavy current applications. Author or co-author of more than 180 papers. Wide tutorial activities at Faculties of Electrical Engineering in Prague and Plzeň.

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Assoc. Prof. Bohuš Ulrych (1937) has been working for a long time in the Department of Theory of Electrical Engineering at the Faculty of Electrical Engineering of UWB in Plzeň. His professional interests are aimed at modern numerical methods of solution of electromagnetic and coupled problems. Author and co-author of about 160 papers and several textbooks. Author of a lot of user's SW for calculation of electromagnetic fields and coupled problems.