TRACE TRANSFORM AND KLT BASED INVARIANT FEATURES AND IMAGE RECOGNITION SYSTEM

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SUMMARY

An important problem in invariant image recognition is automatic extraction of invariant features, which are able to describe an object invariant under translation, rotation and scale. Elegant solution of this problem is using Trace transform based on Triple Features. To compute invariant Triple Features a sophisticated combination of three functionals (Trace, Circus and Diametric) is needed. The paper overline the properties of the functionals must have in order to extract invariant features. The problem of high dimensionality and selecting of optimal combinations of functionals is solved using Karhunen – Loeve transform (KLT). The new architecture of invariant image recognition system based on Trace transform and KLT is presented. The system performance was tested in image recognition experiments.

Keywords: *Trace transform, KLT, functionals, invariant feature extraction, invariant image recognition system*

1. INDRODUCTION

The area of invariant object recognition is widely branch-up in last decade. Much attention has been paid to using transform methods based on well-known rapid, circular, Hough, Radon and Trace transforms [1-8]. It is interesting to note that the Hough, Radon and Trace transform make a family of related transforms. The well-known Radon transform [15], which has found significant applications in computer tomography, astrophysics and computer vision is based on computation of integral of the image function – along lines criss-crossing its domain [4,5,16]. Hough transform [17,18] is a derivative of the Radon transform, which is a variation appropriate for sparse images like edge maps. The Trace transform [19,20,21] is a generalization of the Radon transform in such a way that instead of computing integral along the lines tracing image there are computed any functionals of the image function. Different functionals may be used to produce different Trace transforms from the same image function. A Trace transform of the image is a 2D function of parameters of each tracing line and depends on the choosen functional (Trace functional). Applying two others functionals (Circus and Diametric) to the each parameter of tracing line, yields to a number – Triple feature, which characterize the original image [19-28].

In this paper we overline the properties of the functionals must have in order to extract invariant features. Digital extraction of Triple Features is described, resulting in effective computation of the Trace transform using LUT (Look up Table). The problem of automatic generation of effective invariant Triple Features is solved using Karhunen – Loeve Transform (KLT). New Trace transform and KLT based invariant image recognition system architecture is presented. The system is extended to the colour image processing. Finally system performance was tested in image recognition experiments.

2. TRACE TRANSFORM

The Trace transform [19] can be understood as generalization of the well-known Radon transformation [15,16]. The Radon transform of a real image function $f(x,y)$ defined on Euclidean plane is a function $p(r,\theta)$ defined by computing integrals of $f(x,y)$ along the group of all lines $L(r,\theta)$

$$p(r,\theta)=\iint_D f(x,y)\,\delta(r-x\cos\theta-y\sin\theta)\,dxdy \quad (1)$$

where $r=x\cos\theta+y\sin\theta$ is the normal parameterization of the project line $l(r,\theta,t)$, r is the length of the normal vector from the project line, θ is the angle between the normal vector and x-axis, t is parameter of points along the line, D is the area of support $f(x,y)$ and δ is Dirac distribution (Fig. 1).

The Trace transform is a generalization of the Radon transform in such a way that it computes functional $\mathbf{T}$ (Trace functional) over parameter t along the line $l(r,\theta,t)$, which is not necessarily the integral. Consider scanning of image $f(x,y)$ with lines $l(r,\theta,t)$ in all directions. Denote by $L(r,\theta)$, the set of all lines. The Trace transform is a function $g(\mathbf{T},f,r,\theta)$ defined on $L(r,\theta)$ with the help of Trace functional $\mathbf{T}$ (some functional of the image function $f(x,y)$ when it is considered along the line $l(r,\theta,t)$ as a function of parameter t)

$$g(\mathbf{T},f,r,\theta)=\mathbf{T}\left[f(r,\theta,t)\right] \tag{2}$$

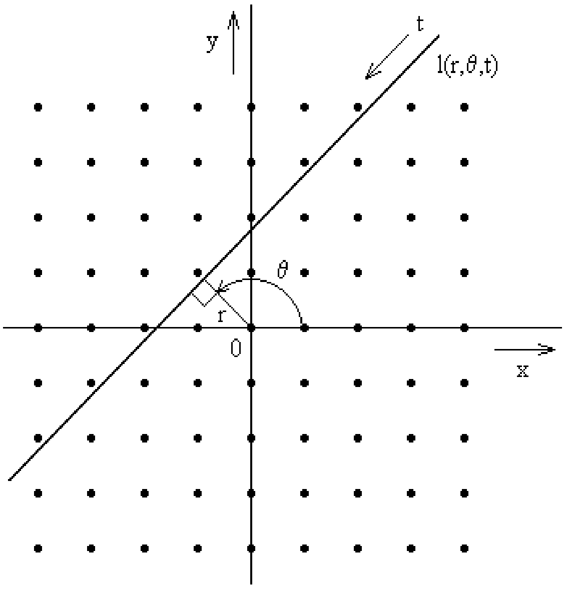

Fig. 1 Definition of the parameters of an image $f(x,y)$ tracing line $l(r,\theta,t)$

We can note that the Trace transform may depend also on the coordinate system used for the image and tracing line set (we consider that they are equal), so $f(r,\theta,t)$ are the values of the image function $f(x,y)$ along the choosen line $l(r,\theta,t)$. Computing functional $\mathbf{T}$, we eliminate parameter t. The result is a 2D function $g(\mathbf{T},f,r,\theta)$ of variables r, θ, which can be interpreted as a new representation of the image $f(x,y)$ defined on the set of $L(r,\theta)$.

3. TRIPLE FEATURE

Triple feature is a number which characterize an image $f(x,y)$ with the help of two additional functionals called Diametric $\mathbf{R}$ and Circular $\boldsymbol{\theta}$ functional. So the Triple feature is defined as [19,24]:

$$\Pi[f]=\boldsymbol{\theta}\left[\mathbf{R}\left[\mathbf{T}\left[f(r,\theta,t)\right]\right]\right] \tag{3}$$

where Π represents extracted Triple feature of image $f(x,y)$ in image space I.

The properties of extracted Triple features depends strongly not only on the processed image $f(x,y)$ (image content), but also on the chosen functionals $\mathbf{T}$, $\mathbf{R}$ and $\boldsymbol{\theta}$. For practical applications these functionals may be choosen, so that the calculated number – Triple feature has one of the following properties [19,22,23,24]:

1. is invariant to translation, rotation and scaling;
2. is sensitive to translation, rotation and scaling so that these parameters for two modified images can be recovered;
3. correlates well with some desired property which we want to identify in a sequence of images.

4. FUNCTIONALS

The traditional mathematical purpose of a functional is to characterize a function by a number. A functional F is an operation defined on a set of functions and resulting in numbers. Let $f(x)$ denote a function of variable $x \in \mathbb{R}$ (set of real numbers). Then the result of applying functional F to function $f(x)$ is denoted

$$\mathsf{F}\left[f(x)\right]=y \tag{4}$$

where y is a single number. Generally functionals have several properties, from the point of extraction of invariant Triple features (how to choice of the three functionals $\mathbf{T}$, $\mathbf{R}$ and $\boldsymbol{\theta}$) it is essential to define following classes of functionals [19,23,24]:

1. <u>Invariant functionals:</u>
   A functional F is called translation invariant if

$$\mathsf{F}\left[f(x+a)\right]=\mathsf{F}\left[f(x)\right] \ , \ \forall a \in \mathbb{R} \tag{5}$$

Translation invariance means that the value of the functional does not change if the function variable shifts. Examples of invariant functionals [19,23,24] are the integral, the median value, the maximal value, the variance of the function, the length of segments, the number of segments, etc. (Tab. 1).

2. <u>Sensitive functionals:</u>
   A functional Z is called sensitive if

$$\mathsf{Z}\left[f(x+a)\right]=\mathsf{Z}\left[f(x)\right]-a \ , \ \forall a \in \mathbb{R} \tag{6}$$

Sensitive functionals may be defined also for periodic function $p(x)=p(x+\tau)$, where τ is a period of the function. A functional is called τ sensitive if

$$\mathsf{Z}\left[p(x+a)\right]=\mathsf{Z}\left[p(x)\right]-a_{(\mathrm{mod}\,\tau)} \ , \ \forall a \in \mathbb{R} \tag{7}$$

3. <u>Homogenous functionals:</u>

a) For invariant functionals we can define abscissa homogeneity property

$$\mathsf{F}\left[f(bx)\right]=b^{k_{\mathsf{F}}}\mathsf{F}\left[f(x)\right] \ , \ \forall b>0 \tag{8}$$

or ordinate homogeneity property

$$\mathsf{F}\left[c.f(x)\right]=b^{\lambda_{\mathsf{F}}}\mathsf{F}\left[f(x)\right] \ , \ \forall c>0 \tag{9}$$

Name	Definition	k	λ
$\mathbf{F}_1$	$\int f(x)\,dx$	-1	1
$\mathbf{F}_2$	$\left(\int \lvert f(x)\rvert^q\,dx\right)^r$	$-r$	qr
$\mathbf{F}_3$	$\int \lvert f'(x)\rvert\,dx$	0	1
$\mathbf{F}_4$	$\int (x-\mathbf{F}_1)^2 f(x)\,dx$	-3	1
$\mathbf{F}_5$	$(\mathbf{F}_4/\mathbf{F}_1)^{1/2}$	-2	0
$\mathbf{F}_6$	$\max\{f(x)\}$	0	1
$\mathbf{F}_7$	$\mathbf{F}_6-\min\{f(x)\}$	0	1
$\mathbf{F}_8$	Amplitude of 1st harmonic of $f(x)$	-	1
$\mathbf{F}_9$	Amplitude of 2nd harmonic of $f(x)$	-	1
$\mathbf{F}_{10}$	Amplitude of 3rd harmonic of $f(x)$	-	1
$\mathbf{F}_{11}$	Amplitude of 4th harmonic of $f(x)$	-	1
$\mathbf{F}_{12}$	Length of the minimum segment		
$\mathbf{F}_{13}$	Number of segments		
$\mathbf{F}_{14}$	$\mathrm{var}\{f(x)\}$		
$\mathbf{F}_{15}$	Hilbert norm $\left(\int f^2(x)\,dx\right)^{1/2}$	$-1/2$	1
$\mathbf{F}_{16}$	Number of extremes of the $f(x)$		

Tab. 1 Invariant functionals

Name	Definition	c	λ
$\mathbf{Z}_1$	$\left\{\int x.f(x)\,dx\right\}/\mathbf{F}_1$	-1	0
$\mathbf{Z}_2$	$\left(\int \lvert f(x)\rvert^q\,dx\right)^r$	$-r$	qr
$\mathbf{Z}_3$	$\int \lvert f'(x)\rvert\,dx$	0	1
$\mathbf{Z}_4$	$\int (x-\mathbf{F}_1)^2 f(x)\,dx$	-3	1
$\mathbf{Z}_5$	$(\mathbf{F}_4/\mathbf{F}_1)^{1/2}$	-2	0
$\mathbf{Z}_6$	$\max\{f(x)\}$	0	1
$\mathbf{Z}_7$	$\mathbf{F}_6-\min\{f(x)\}$	0	1
$\mathbf{Z}_8$	Amplitude of 1st harmonic of $f(x)$	-	1
$\mathbf{Z}_9$	Amplitude of 2nd harmonic of $f(x)$	-	1

Tab. 2 Sensitive functionals

Functional	**Homogeneity constant**	
	k	λ
$\mathbf{F}_1\ \mathbf{F}_2$	$k_{\mathbf{F}_1}+k_{\mathbf{F}_2}$	$\lambda_{\mathbf{F}_1}+\lambda_{\mathbf{F}_2}$
$(\mathbf{F})^q$	$q\ k_{\mathbf{F}}$	$q\ \lambda_{\mathbf{F}}$
$\mathbf{F}_1/\mathbf{F}_2$	$k_{\mathbf{F}_1}-k_{\mathbf{F}_2}$	$\lambda_{\mathbf{F}_1}-\lambda_{\mathbf{F}_2}$

Tab. 3 Functionals with desired homogeneity constants

These properties are good to work with scaled images. Functionals do not necessarily obey these properties. In practice used functionals have such properties or they can be easily modified to acquire them. The constants k_{F} and λ_{F} are called homogeneity constants of the functional.

b) For Sensitive functionals may be necessary the following property

$$\mathsf{Z}\left[f(bx)\right]=\frac{1}{b}\mathsf{Z}\left[f(x)\right] \quad , \quad \forall b\in\mathbb{R} \tag{10}$$

so the scaling the independent variable scales the results inversely. Using definition (6) results

$$\mathsf{Z}\left[f(bx+a)\right]=\mathsf{Z}\left[f(bx)\right]-a=\frac{1}{b}\mathsf{Z}\left[f(x)\right]-a \tag{11}$$

and

$$\mathsf{Z}\left[f(b(x+a))\right]=\frac{1}{b}\mathsf{Z}\left[f(x+a)\right]=\frac{1}{b}\left(\mathsf{Z}\left[f(x)\right]-a\right) \tag{12}$$

Thus abscissa homogeneity constant for sensitive functionals is $k_{\mathsf{Z}}=-1$. Sensitive functionals may have ordinate homogeneity property

$$\mathsf{Z}\left[c.f(x)\right]=\mathsf{Z}\left[f(x)\right] \quad , \quad \forall c>0 \tag{13}$$

Thus the ordinate homogeneity constant for sensitive functionals is $\lambda_{\mathsf{Z}}=0$.

The critical parameters that characterize an Invariant functional F are homogeneity constants k_{F} and λ_{F}. We can construct new functionals of a desired value of homogeneity constants using combination of functionals according to Table 3.

5. INVARIANT TRIPLE FEATURES

To extract invariant Triple features (i.e. features invariant to translation, rotation and scaling) we choose functionals **T**, **R** and **θ** with the following properties [19,22,24,25,26,27]:

Combination I.:

a) Trace functional **T** is translation invariant (5) with abscissa homogeneity constant k_T (8);

b) Diametric functional **R** is translation invariant (5) with abscissa and ordinate homogeneity constant k_R (8) and λ_R (9);

c) Circular functional **θ** is translation invariant (5) with abscissa and ordinate homogeneity constant k_θ and λ_θ;

Then Triple features computed for original f and geometrical modified image f_M are related by the formula [13, 18]:

$$\Pi\left[f_M\right]=c^{-\lambda_\theta(k_T\lambda_R+k_R)}\Pi\left[f\right] \tag{14}$$

where c is the scaling factor between the two images f and f_M.

If there is no scale difference between the images f and f_M then from (14) results

$$\Pi\left[f_M\big|_{c=1}\right]=\Pi\left[f\right] \tag{15}$$

so for creation of invariant Triple feature any combination of invariant functionals can be used.

If there is scale difference (i.e. $c\neq 1$) then the condition for invariance of Triple feature is

$$\lambda_\theta\left(k_T\lambda_R+k_R\right)=0 \tag{16}$$

Combination II.:

a) Trace functional **T** is translation invariant (5) with abscissa homogeneity constant k_T;

b) Diametric functional **R** is sensitive with properties (6) and (7), homogeneity constants are: $k_R=-1$ and $\lambda_R=0$;

c) Circular functional **θ** is translation invariant (5) with ordinate homogeneity constant λ_θ, and is not sensitive to the first harmonic of the function f.

Then the Triple features computed for original f and geometricaly modified image f_M are related by the formula [19,24]:

$$\Pi\left[f_M\right]=c^{\lambda_\theta}\Pi\left[f\right] \tag{17}$$

and condition for creation invariant Triple feature is

$$\lambda_\theta=0 \tag{18}$$

Combinations I. and II. can be represented in single form

$$\Pi[f_M]=c^{\alpha}\Pi[f] \tag{19}$$

Where

$$\alpha=\begin{cases}-\lambda_\theta\left(k_T\lambda_R+k_R\right) & \text{for combination I.}\\ \lambda_\theta & \text{for combination II.}\end{cases} \tag{20}$$

If $\alpha=0$ then the computed Triple feature is invariant to translation, rotation and scaling of the image.

The condition $\alpha=0$ is too restrictive, so in practice are considered Triple features with $\alpha\neq 0$ [13,17,18,19,20,21]. Since we choose the functionals **T**, **R** and **θ** with known properties, so the α is known. Then every Triple feature we compute can be normalized in the form

$$\Pi_n\left[f\right]=\left|\Pi\left[f\right]\right|^{1/\alpha}\ \operatorname{sign}\left\{\Pi\left[f\right]\right\} \tag{21}$$

and formula (19) can be simplified to the form

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$$\Pi_n\left[f_M\right] = c^{-1}\Pi_n\left[f\right] \tag{22}$$

so the computed normalized Triple feature depends linearly on the scaling factor c of the modified image. If we consider the ratio of two normalised Triple features

$$\Pi_r = \frac{\Pi_n^{(1)}}{\Pi_n^{(2)}} \tag{23}$$

The resulting Triple feature is invariant to translation, rotation and scaling of the image (Tab. 4).

The process of feature generation may be most robust if we instead of using not only two normalised Triple features, but compute a set of such a features $\Pi_n^{(i)}, i = 1,\ldots,N$. The features may be considered as a feature vector

$$\mathbf{S} = \left(\Pi_n^{(1)},\ldots,\Pi_n^{(N)}\right) \tag{24}$$

The norm of feature vector is directly proportional to the scaling factor c. The direction of the vector **S** in feature space is invariant to translation, rotation and scaling of the image. As an invariant feature vector can be used the direction coefficients of this vector in the feature space

$$\mathbf{S} = \frac{\Pi_n^{(i)}}{\sqrt{\sum_{j=1}^{n}\left(\Pi_n^{(j)}\right)^2}} \tag{25}$$

$\Pi_n^{(1)}$			$\Pi_n^{(2)}$		
T	**R**	**θ**	**T**	**R**	**θ**
$\mathbf{F}_2$	$\mathbf{F}_1$	$\mathbf{F}_{11}$	$\mathbf{F}_2$	$\mathbf{F}_1$	$\mathbf{F}_9$
$\mathbf{F}_5$	$\mathbf{F}_1$	$\mathbf{F}_{11}$	$\mathbf{F}_5$	$\mathbf{F}_1$	$\mathbf{F}_9$
$\mathbf{F}_5$	$\mathbf{F}_2$	$\mathbf{F}_{10}$	$\mathbf{F}_5$	$\mathbf{F}_3$	$\mathbf{F}_3$
$\mathbf{F}_4$	$\mathbf{F}_4$	$\mathbf{F}_9$	$\mathbf{F}_4$	$\mathbf{F}_4$	$\mathbf{F}_{11}$
$\mathbf{F}_4$	$\mathbf{F}_4$	$\mathbf{F}_7$	$\mathbf{F}_4$	$\mathbf{F}_4$	$\mathbf{F}_7$

Tab. 4 Some invariant Triple Feature functional combinations

6. DIGITAL EXTRACTION OF TRIPLE FEATURES

Input digital image $\mathbf{F}(i,j)$, $i = 0,2,\ldots,N_x - 1$ and $j = 0,2,\ldots,N_y - 1$ is a matrix representation of 2D sampled continuous image function $f(x,y)$ on a Cartesian grid (Fig. 2) in (x,y) space. The steps Δx, Δy are properly choosen according to Shannon sampling theorem [29].

For simple digital computation of Trace transform is necessary sampling the line set , i.e. the parameters $r = k.\Delta x$, $k = 0,1,\ldots,N_r - 1$, $\theta = l.\Delta\theta$, $l = 0,1,\ldots,N_\theta - 1$ and $t = m.\Delta t$, $m = 0,1,\ldots,N_t - 1$. From the practical point of view we choose $N_x = N_y$ and $\Delta x = \Delta y$. For digitalization of parameter r we choose the same discretization steps as for axis *x* and *y* so

$$\Delta r = \Delta x = \Delta y \tag{26}$$

and for $\theta = \pi/4$

$$N_r = \operatorname{int}\left[\sqrt{2}N_x\right] \tag{27}$$

so $r = k.\Delta x$, $k = 0,1,\ldots,N_r - 1$.

The discretization steps for parameter θ we choose as (Fig. 2)

$$\Delta\theta = \arctan\frac{\Delta y}{\frac{N_x}{2}\Delta x} = \arctan\frac{2}{N_x} \tag{28}$$

and

$$N_\theta = \operatorname{int}\left[\frac{\pi}{\Delta\theta}\right] \tag{29}$$

so $\theta = l.\Delta\theta$, $l = 0,\ldots,N_\theta - 1$.

The discretization step for parameter t we choose

$$\Delta t = \Delta r = \Delta x = \Delta y \tag{30}$$

so $t = m.\Delta t$, $m = 0,1,\ldots,N_t - 1$. The number of steps in parameter t depends on line parameters (k,l). The maximum number of t parameter steps is for line with parameter $r = 0$ and $\theta = \pi/4$

$$N_t(\max) = \operatorname{int}\left\{\sqrt{2}N_x\right\} \tag{31}$$

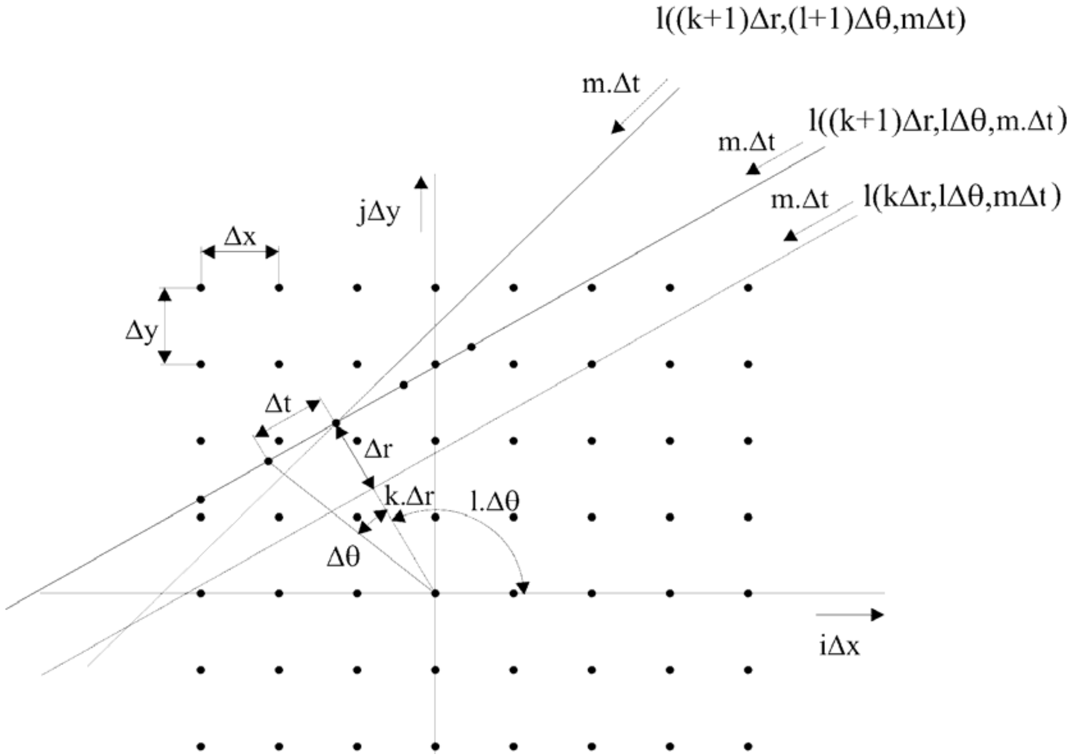

Fig. 2 Discretization of r, θ and t parameters

For digital computation of Trace transform in the point **P** of digital tracing line $l\left(k\Delta r, l\Delta\theta, m\Delta t\right)$ (Fig. 3) it is necessary to resample the digital input image $\mathbf{F}\left(i,j\right)$. There are 4 neighbours points (Fig. 3) in Cartesian grid $\mathbf{A}_i, i = 1,2,3,4$, for which we are able to extract exact values of the input image function $f\left(x,y\right)$. The exact value of image function $f\left(x,y\right)$ in the point **P** may be computed by interpolation from the corresponding elements of the matrix $\mathbf{F}\left(i,j\right)$. For the maximal computational simplicity we use an approximation of exact value of function $f\left(x,y\right)$ in point **P** as the value of $\mathbf{F}\left(i,j\right)$ at the nearest neighbour point in Cartesian grid (i.e. at point A_2 in Fig. 3). Mapping from 2D Cartesian grid $\mathbf{F}\left(i,j\right)$ to 3D matrix $\mathbf{S}\left(k,l,m\right)$ of points **P** on digital sampling lines may be pre-

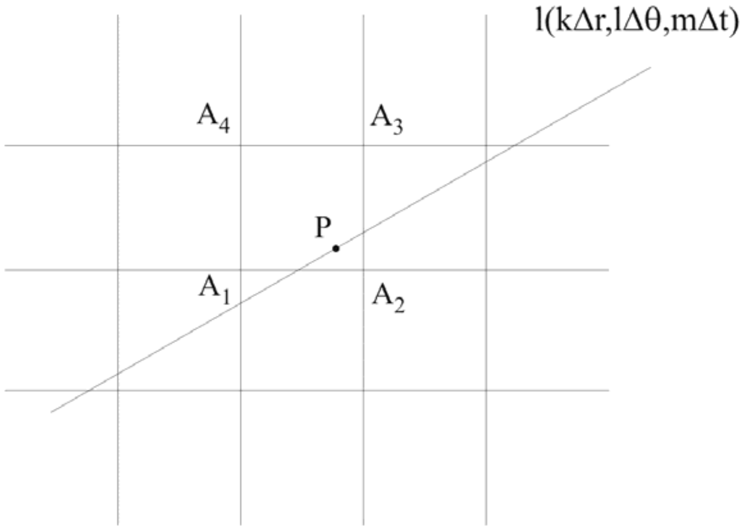

Fig. 3 Four neighbours points of point **P** on digital tracing line $l\left(k\Delta r, l\Delta\theta, m\Delta t\right)$ in Cartesian grid

computed (Fig. 4) for the speeding of the computation of the Trace transform for a known dimensions of input digital images $N_x \times N_y$ pixels (in practice 64×64, 128×128, 256×256, 512×512 or 1024×1024 pixels). Then the digital version of the choosen Trace functionals is computed on 3D matrix $\mathbf{S}(k,l,m)$ which results on computation of digital Trace transform represented in the form of Trace matrix $\mathbf{G}(k,l)$, $k = 0,1,\ldots,N_r - 1$, $l = 0,\ldots,N_\theta - 1$. After computing digital Diametric $\mathbf{R}$ and Circular $\boldsymbol{\theta}$ functionals we obtain Triple feature

$$\Pi[f] = \boldsymbol{\theta}\big[\mathbf{R}[\mathbf{G}]\big] \tag{32}$$

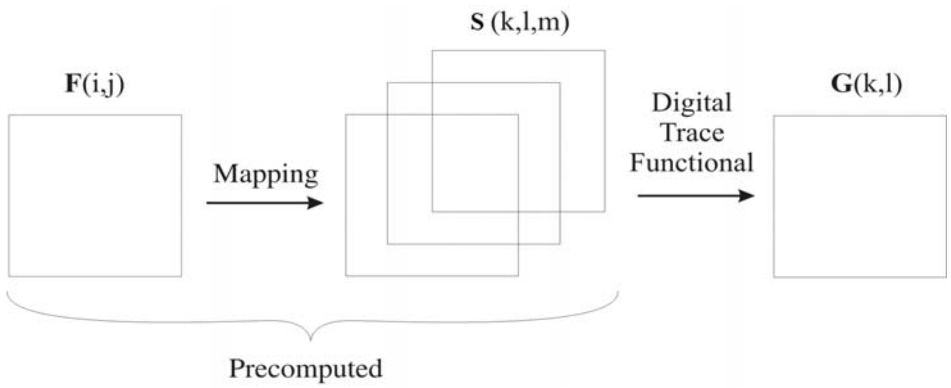

Fig. 4 Digital computation of Trace transform

7. AUTOMATIC GENERATION OF EFFECTIVE INVARIANT FEATURES

We assume that there are M number of image classes for recognition, and n_i number of sample images for i-th image class. The feature vector $\mathbf{S}_\mathbf{j}^\mathbf{i}$ of each sample image can be calculated using equation (25), where $i = 1,\ldots,M$ and $j = 1,\ldots,n_i$. Than we can compute the following characteristics of the recognition problem:

a) Average feature vector $\overline{\mathbf{S}^\mathbf{i}}$ for class i:

$$\overline{\mathbf{S}^\mathbf{i}} = \frac{1}{n_i}\sum_{j=1}^{n_i}\mathbf{S}_\mathbf{j}^\mathbf{i} \tag{33}$$

b) Average feature vector $\overline{\mathbf{S}}$ for all classes:

$$\overline{\mathbf{S}} = \frac{1}{M}\sum_{i=1}^{M}\overline{\mathbf{S}^\mathbf{i}} \tag{34}$$

c) Variance of feature vector $\mathbf{var}(\mathbf{S}^\mathbf{i})$ for class i:

$$\mathbf{var}(\mathbf{S}^\mathbf{i}) = \frac{1}{n_i}\sum_{j=1}^{n_i}(\mathbf{S}_\mathbf{j}^\mathbf{i} - \overline{\mathbf{S}^\mathbf{i}}) \tag{35}$$

d) Variance of feature vector $\mathbf{var}(\mathbf{S})$ for all classes:

$$\mathbf{var}(\mathbf{S}) = \frac{1}{M}\sum_{i=1}^{M}\mathbf{var}(\mathbf{S}^\mathbf{i}) \tag{36}$$

e) Covariance matrix $\mathbf{C}$ for all training image features:

$$\mathbf{C} = \frac{1}{M}\sum_{i=1}^{M}(\overline{\mathbf{S}^\mathbf{i}} - \overline{\mathbf{S}})(\overline{\mathbf{S}^\mathbf{i}} - \overline{\mathbf{S}})^T \tag{37}$$

The effectively selected features should have small intraclass variance and high interclass variance. Note that not all the elements of the feature vector are independent of other elements, so we can consider reducing the redundant information contained in them. Here the principal component analysis based on Karhunen – Loeve transform (KLT) can be used [30,31,32].

The size of $\overline{\mathbf{S}}$ is N, so the size of matrix $\mathbf{C}$ is NxN. Since $\mathbf{C}$ is a symmetric matrix of real values, it has N real eigenvalues $\lambda_i, i = 1,...,N$ and the corresponding eigenvectors $\mu_i, i = 1,...,N$. Note that calculated eigenvalues λ_i are sorted in decreasing order, so $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_N$. For data compression, i.e. decreasing of feature vector dimension P eigenvalues $(P < N)$ are taken out. Computing the correspondent eigenvectors KLT based feature decorrelation transform matrix $\mathbf{D}$ is created

$$\mathbf{D} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{pmatrix} \tag{38}$$

Then decorrelated features can be computed using formula

$$\mathbf{v}^\mathbf{i} = \mathbf{D}(\mathbf{S}^\mathbf{i} - \overline{\mathbf{S}}) \tag{39}$$

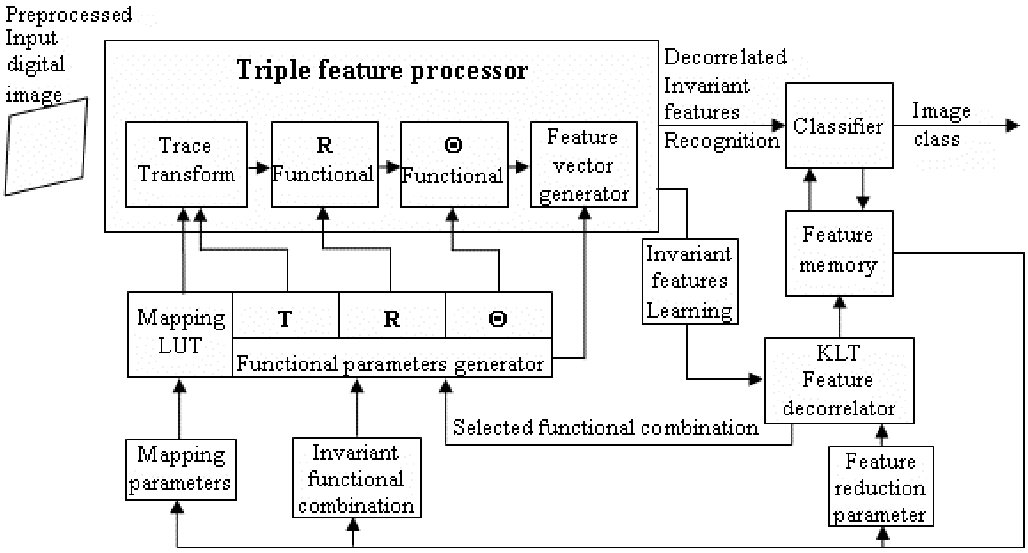

Fig. 5 Architecture of invariant image recognition system

8. INVARIANT IMAGE RECOGNITION SYSTEMS

Trace transform and KLT based invariant image recognition system architecture is on Fig. 5. The system contains 5 blocks: Triple feature processor (with Feature vector generator), Functional parameters generator, KLT feature decorrelator, Feature memory and Classifier. The Triple feature processor automatically generates Trace transform based invariant features for the processed image based on mapping parameters, choosen combination of **T**, **R**, **θ** (invariant functional combinations) and their parameters. Generated invariant feature vectors are in Learning mode of the system processed with KLT feature decorrelator, optimal feature vectors are feeded to the Feature memory for each Image class. In Recognition mode, Feature vector is input to the Classifier and is compared with stored reference Feature vectors and decision of Image class is made. We use simple Euclidean classifier. Recognition efficiency of the system is used in feedback to optimise Feature reduction parameters, Invariant functional combinations and mapping parameters of the system.

The proposed system was realised as a software tool using C++ on a powerful multimedia PC extending our previous software tools [33-44]. Can be run on OS Windows 2k, XP. Memory requirements are 2GB. Required time for Learning mode and recognition procedure depends on used processor, recognition image class, memory type, etc. (typical is less than μs).

The colour scheme of processed images should be binary, grey and coloured. During recognition process of colour images y,u,v colour model is used [39,40,41]. In Colour preprocessing (Fig. 6), dominant colours computed from histogram (based on u,v, values) are computed for fast image sorting based on colour structure or distribution of recognized images. The Trace transform and KLT based invariant features are computed from y values.

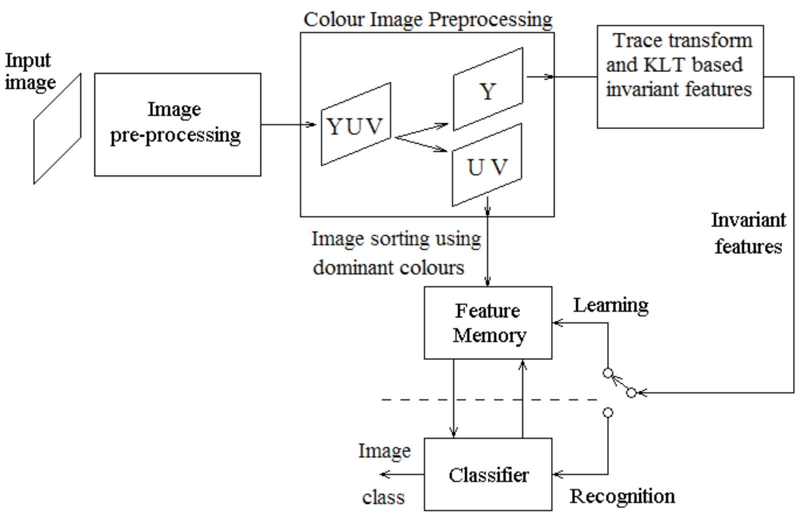

Fig. 6 Colour image processing scheme

9. EXPERIMENTS

The proposed automatic invariant feature extraction system has been tested on different image classes: coloured road signs, binary symbols, coloured plates and aircraft. In experiments original (Fig. 7) and modified (shifted, rotated and scaled) images are used (Fig. 8). The results for aircraft images recognition are presented. In the aircraft recognition experiment had been used more than 1500 modified images (shifted, rotated, scaled and corrupted with Gaussian noise). The size of original Feature vector is more than 100 elements and is using KLT decorrelation method decreased to 9 with recognition efficiency rate 0,99 – 0,87 (depending on the modification of the original aircraft images).

As can be noted that for a ceirtan image resolution, for the choosen class of invariant functional combinations **T**, **R**, **θ** and choosen image class the mapping LUT, functional parameters and KLT transform matrix (i.e. optimal feature selection) can be precomputed (i.e. calculated off-line). The resulting optimal feature vector size is very small, the classification process may be easy and fast.

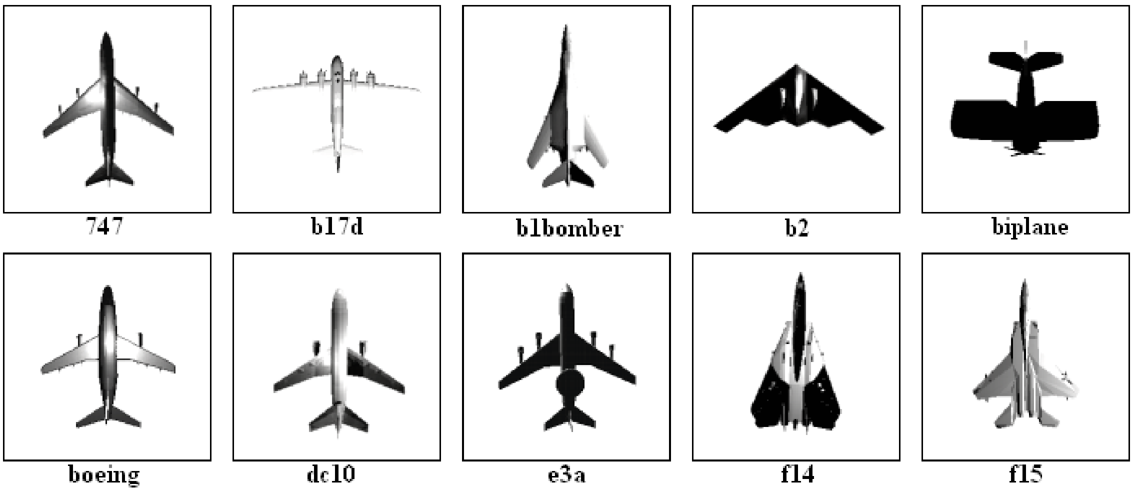

Fig. 7 Aircraft images

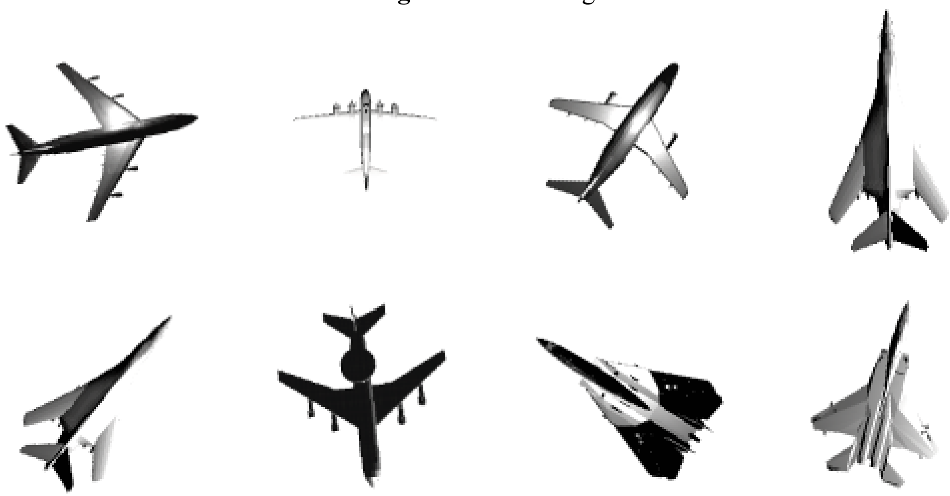

Fig. 8 Samples of modified aircraft images

10. CONCLUSIONS

Trace transform and KLT based automated invariant extraction method is suitable to extract many mathematical invariant (translation, rotation and scale) features of an image. We implemented the proposed elegant invariant feature extraction method as a programme package based on automatic KLT based selection of appropriate T, R and **θ** functionals. The selected features are suitable to characterize many object classes in invariant image recognition systems.

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