TIME-DEPENDENT QUEUE-SIZE DISTRIBUTION IN THE FINITE $GI/M/1$ MODEL WITH AQM-TYPE DROPPING

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ABSTRACT

A $GI/M/1$-type queueing system with finite buffer capacity and AQM-type packet dropping is investigated. Even when the buffer is not saturated an incoming packet can be dropped (lost) with probability dependent on the instanteneous queue size. The system of integral equations for time-dependent queue-size distribution conditioned by the number of packets present in the system initially is built using the embedded Markov chain approach. The solution of the corresponding system written for Laplace transforms is found using the linear algebra. Numerical examples, in which different-type dropping functions are investigated in some network-motivated traffic scenarios, are attached as well.

Keywords: *AQM (Active Queue Management), dropping function, finite buffer, queue size, time-dependent distribution.*

1. PRELIMINARIES

The phenomenon of packet losses is a typical one in packet-oriented networks like e.g. the Internet. Obviously, due to finite capacities of IP routers' buffers, the queue of packets waiting for processing can not be unbounded. In consequence, during the buffer overflow period all the incoming packets are naturally lost (Tail Drop algorithm). Unfortunately, such a policy has different disadvantages. For example, it is difficult to stabilize the arrival intensity on the proper level and hence many retransmissions are necessary. The Active Queue Management (AQM), in the contrast to Passive Queue Management (PQM), based on the idea of Tail Drop, allows for dropping the arriving packets even when the buffer is not completely saturated. The dropping probability can depend on the mean or instanteneous queue size. In consequence the reduction of the buffer queue length is being obtained in two diffrent ways:

- by immediate deleting the incoming packet via dropping function (short-term reduction);
- by decreasing the intensity of arrivals as a reaction of the source host for packet dropping, according to TCP/IP protocol requirements (long-term reduction).

In [8] the first model with AQM-type packet dropping was introduced, with a linear dropping function. Looking for the mathematical description of the packet dropping function being optimal with respect to one or more criteria, resulted in many papers in which some other shapes of dropping functions were proposed and investigated. The idea of random exponential marking of packets which should be dropped (REM-type dropping) can be found in [3] and [15]. In [16] a doubly-linear (broken line) dropping function is considered and in [7] a modification of the classical RED algorithm towards traffic conditions adaptation (ARED-type dropping) can be found. Despite the fact that different AQM-type queueing models were proposed, they were not investigated analitycally sufficiently well. A compact-form representation for the stationary queue-size distribution in the system with packet dropping and Poisson arrivals was obtained in [5]. A direct approach to the study of the $M/G/1$-type finite-buffer queue with packet dropping can be found in [13]. In [9] the steady-state characteristics of the AQM-type finite-buffer $M/M/1$ system with single and batch arrivals were derived, namely probability distributions of the queue-size, number of consecutively lost packets and time between two successive packets accepted for service. The results from [9] were generalized in [18] and [19] for the model with Poisson input stream in which the arriving packets have generally distributed volumes and the total volume of the system is finite. The case of the multi-server system with AQM was analyzed in [20] and the compact-form representation for the stationary queue-size distribution was obtained there.

In this article an algebraic method for computing time-dependent queue-size distribution in the finite-buffer model with general-type independent input stream and packet dropping is proposed. There are at least two main motivations for such a study. The first one is that in the literature the results concerning models with AQM are obtained mainly for the equilibrium. The next is that they are often restricted to the case of Poisson (or compound Poisson) arrival process.

Transient results for the finite $GI/M/1$-type queueing models can be found e.g. in [10], [11] and [14]. Compact-form formulae for the non-stationary queue-size distribution for some infinite-buffer models were obtained e.g. in [4] and [12].

The remaining part of the paper is organized as follows. In the next Section 2 the system of integral equations for the time-dependent queue-size distributions, conditioned by the number of packets at $t = 0$, is obtained, using the approach based on the idea of embedded Markov chain and the law of total probability. The solution of the corresponding system built for the Laplace transforms is found in algebraic form in Section 3. Numerical examples are contained in the last Section 4, where different dropping scenarios (analytical formulae for dropping functions) are considered.

2. SYSTEM OF EQUATIONS FOR TIME-DEPENDENT QUEUE-SIZE DISTRIBUTION

Let us take into consideration a finite-buffer queueing model with general-type independent input stream, described by the distribution function $F(\cdot)$ of interarrival times, and exponentially distributed service times with mean μ^{-1}. The capacity of the buffer queue equals $K-1$, so the number of packets in the system is bounded by K (we have $K-1$ places in the buffer and one place in service). The arriving process of incoming packets is controlled via the general-type dropping function $d(\cdot)$ that for fixed n takes on the value $d(n) = d_n$ being the probability that the entering packet finding the system in state n will be dropped. Evidently, $d_0 \geq 0$ and $d_K = 1$.

Define the conditional queue-size distribution in the system as follows:

$$q_n(t,m) = \mathbf{P}\{X(t) = m \,|\, X(0) = n\}, \tag{1}$$

where $0 \leq m, n \leq K$ and $X(t)$ stands for the number of packets present in the system at time t.

It is easy to note that if the system begins the operation being empty ($X(0) = 0$), then the following integral equation holds true:

$$q_0(t,m) = \int_0^t \Big[(1-d_0)q_1(t-x,m) + d_0 q_0(t-x,m)\Big] dF(x) + \big(1-F(t)\big)\delta_{m,0}, \tag{2}$$

where $\delta_{i,j}$ denotes the Kronecker delta function.

Let us comment (2) briefly. Indeed, if the first packet enters at time $x < t$ then the system continues the operations beginning with time x with exactly one packet present with probability $1-d_0$ (if the arriving packet will be "accepted" for service), or being still empty with probability d_0 (if the arriving packet will be dropped). Of course, if the first arrival occurs after t the random event $\{X(t) = m\}$ is equivalent to $\{m = 0\}$.

Investigate now the case of the buffer being non-empty and non-saturated primarily. By virtue of the fact that the arrival epochs are Markov moments in the $GI/M/1$-type queue, and using the continuous version of the law of total probability with respect to the first arrival moment, we get the following system of Volterra-type integral equations:

$$\begin{aligned} &q_n(t,m) \\ &= \int_0^t \Bigg[\sum_{k=0}^{n-1} \frac{(\mu x)^k}{k!} e^{-\mu x}\Big((1-d_{n-k})q_{n-k+1}(t-x,m) \\ &+ d_{n-k}q_{n-k}(t-x,m)\Big) \\ &+ \sum_{k=n}^{\infty} \frac{(\mu x)^k}{k!} e^{-\mu x}\Big((1-d_0)q_1(t-x,m) \\ &+ d_0 q_0(t-x,m)\Big)\Bigg] dF(x) \\ &+ \big(1-F(t)\big)e^{-\mu t}\left(I\{n \geq m \geq 1\}\frac{(\mu t)^{n-m}}{(n-m)!} + \delta_{m,0}\sum_{k=n}^{\infty}\frac{(\mu t)^k}{k!}\right), \end{aligned} \tag{3}$$

where $1 \leq n \leq K-1$ and $I\{\mathbb{A}\}$ is the indicator of the random event $\mathbb{A}$.

The first summand on the right side of (3) relates to the situation in which the first arrival occurs at time $x < t$ and, simultaneously, the buffer is not empty before x. The second summand describes the situation in which before $x < t$ the queue empties, so at x the system "renews" the operation being empty or with one packet present, in dependence on the dropping or acceptance of the packet arriving at time x, respectively. Finally, the last summand on the right side of (3) presents the situation in which there are any arrivals before instant t, at which the queue length is measured.

Note that in the case of buffer being saturated ($n = K$) initially, we have

$$\begin{aligned} &q_K(t,m) \\ &= \int_0^t \Bigg[e^{-\mu x}q_K(t-x,m) \\ &+ \sum_{k=1}^{K-1} \frac{(\mu x)^k}{k!} e^{-\mu x}\Big((1-d_{K-k})q_{K-k+1}(t-x,m) \\ &+ d_{K-k}q_{K-k}(t-x,m)\Big) \\ &+ \sum_{k=K}^{\infty} \frac{(\mu x)^k}{k!} e^{-\mu x}\Big((1-d_0)q_1(t-x,m) \\ &+ d_0 q_0(t-x,m)\Big)\Bigg] dF(x) \\ &+ \big(1-F(t)\big)e^{-\mu t}\left(I\{m \geq 1\}\frac{(\mu t)^{K-m}}{(K-m)!} + \delta_{m,0}\sum_{k=K}^{\infty}\frac{(\mu t)^k}{k!}\right). \end{aligned} \tag{4}$$

In fact, the right side of the equation (4) differs from (3) by the sum taken from $k = 1$, and the component $e^{-\mu x}q_K(t-x,m)$ standing under the integral. Indeed, if the first departure occurs after the first arrival moment x, then the epoch x is "inside" the buffer overflow period, so the packet incoming at x is lost.

3. CORRESPONDING SYSTEM FOR LAPLACE TRANSFORMS

In this section we derive the corresponding system of equations for Laplace transforms of conditional queue-size distributions, write it in a matrix form and next find the representation for solution.

Introduce firstly the following notations:

$$\widetilde{q}_n(s,m) = \int_0^\infty e^{-st} q_n(t,m)\,dt, \tag{5}$$

$$\theta_k(s) = \int_0^\infty e^{-(s+\mu)t}\frac{(\mu t)^k}{k!}dF(t), \tag{6}$$

$$f(s) = \int_0^\infty e^{-st}dF(t), \tag{7}$$

$$\sigma_n(s,m) = \int_0^\infty e^{-(s+\mu)t}\big(1-F(t)\big)\left(\frac{(\mu t)^{n-m}}{(n-m)!}I\{n \geq m \geq 1\} + \delta_{m,0}\sum_{k=n}^{\infty}\frac{(\mu t)^k}{k!}\right), \tag{8}$$

where $\mathrm{Re}(s) > 0$.

Applying (5)–(8) in the system (2)–(4) gives

$$\widetilde{q}_0(s,m) = \big[(1-d_0)\widetilde{q}_1(s,m) + d_0\widetilde{q}_0(s,m)\big] f(s) + \frac{1-f(s)}{s}\delta_{m,0}, \quad (9)$$

$$\widetilde{q}_n(s,m) = \sum_{k=0}^{n-1}\theta_k(s)\big[(1-d_{n-k})\widetilde{q}_{n-k+1}(s,m) + d_{n-k}\widetilde{q}_{n-k}(s,m)\big] + \sum_{k=n}^{\infty}\theta_k(s)\big[(1-d_0)\widetilde{q}_1(s,m) + d_0\widetilde{q}_0(s,m)\big] + \sigma_n(s,m), \quad (10)$$

where $1 \le n \le K-1$, and besides

$$\widetilde{q}_K(s,m) = f(s+\mu)\widetilde{q}_K(s,m) + \sum_{k=1}^{K-1}\theta_k(s)\big[(1-d_{K-k})\widetilde{q}_{K-k+1}(s,m) + d_{n-k}\widetilde{q}_{K-k}(s,m)\big] + \sum_{k=K}^{\infty}\theta_k(s)\big[(1-d_0)\widetilde{q}_1(s,m) + d_0\widetilde{q}_0(s,m)\big] + \sigma_K(s,m). \quad (11)$$

The equations of the system (9)–(11) can be rewritten in the matrix form. Let $\mathbb{A}(s)$ be the matrix of coefficients with $K+1$ rows and $K+1$ columns, and with successive elements defined as follows (it will be convenient for us to number successive rows and columns beginning with 0 instead of 1):

$$a_{0,0}(s) = 1 - d_0 f(s),$$
$$a_{0,1}(s) = (d_0 - 1) f(s),$$
$$a_{0,j}(s) = 0, \quad (12)$$

for $2 \le j \le K$, and

$$a_{i,0}(s) = d_0 \sum_{k=i}^{\infty}\theta_k(s), \ \ 1 \le i \le K, \quad (13)$$

$$a_{i,1}(s) = d_1\theta_{i-1}(s) + (1-d_0)\sum_{k=i}^{\infty}\theta_k(s) - \delta_{i,1}, \quad 1 \le i \le K, \quad (14)$$

$$a_{i,j}(s) = (1-d_{j-1})\theta_{i-j+1}(s) + d_j\theta_{i-j}(s) - \delta_{i,j}, \quad 1 \le i \le K, \ \ 2 \le j \le \min\{i, K-1\}, \quad (15)$$

$$a_{i,i+1}(s) = (1-d_i)\theta_0(s), \ \ 1 \le i \le K-1, \quad (16)$$

$$a_{K,K}(s) = (1-d_{K-1})\theta_1(s) + f(s+\mu) - 1, \quad (17)$$

$$a_{i,j}(s) = 0, \quad \text{otherwise}. \quad (18)$$

Next, let us define the $(K+1)\times 1$ matrix $\mathbb{B}(s,m)$ as

$$\mathbb{B}(s,m) = \big[s^{-1}\big(1-f(s)\big)\delta_{m,0}, -\sigma_1(s,m), ..., -\sigma_K(s,m)\big]^T. \quad (19)$$

Finally, let $\mathbb{Q}(s,m)$ be the $(K+1)\times 1$ column of unknowns i.e.

$$\mathbb{Q}(s,m) = \big[\widetilde{q}_0(s,m), \widetilde{q}_1(s,m), ..., \widetilde{q}_K(s,m)\big]^T. \quad (20)$$

From the definition (1) it follows that the solution of the system (9)–(11) exists and is unique, so the following theorem is true: **Theorem 1.** *The representation for the column $\mathbb{Q}(s,m)$ with entries being Laplace transforms of the conditional transient queue-size distributions in the $GI/M/1$-type queueing system with total capacity K can be found from the formula*

$$\mathbb{Q}(s,m) = \mathbb{A}^{-1}(s)\mathbb{B}(s,m), \quad (21)$$

where $0 \le m \le K$, and the matrices $\mathbb{A}(s)$ and $\mathbb{B}(s,m)$ were defined in (12)–(18) and (19) respectively.

Remark. *From the representation (21), for a completely determined particular model, it is possible to obtain the steady-state distribution q_m by using the well-known Tauberian theorem i.e.*

$$q_m = \lim_{s\downarrow 0} s\widetilde{q}_n(s,m), \quad (22)$$

where n can be chosen arbitrarily from 0 to K, due to the fact that the stationary probability does not depend on the initial condition of the system.

4. NUMERICAL EXAMPLES

In this section we investigate the influence on the analytical shape of a dropping function on transient and stationary queue-size distribution on a certain network-motivated example. Consider a stream of packets of average sizes 800 B arrive at the IP router with rate 120 Mb/s, and the throughput of the output link equals 100 Mb/s, so the system is overloaded ($\rho = 1.2$). Assume that $K = 6$ and that interarrival times have Erlang distributions with two degrees of freedom and with parameter λ. From the traffic characteristics it follows that

$$\lambda = 37500, \quad \mu = 15625.$$

Define the following RED linear dropping function:

$$d_n = \begin{cases} 0, & n \le 1, \\ 0.2(n-1), & 1 < n < 2, \\ 1, & n \ge 2. \end{cases} \quad (23)$$

In Fig. 1 we present time-dependent conditional distributions

$$\mathbf{P}\{X(t) = 1 \,|\, X(0) = n\}$$

for three different initial buffer states: $n = 0, 3$ and 6. To invert the right side of the formula (21) we use the algorithm of numerical Laplace transform inversion introduced in [2], based on Bromwich integral and Euler's summation of the alternating series. As it is easy to observe, all the characteristics stabilize at the stationary probability q_1 after about 0.0006 [s] (0.6 [ms]).

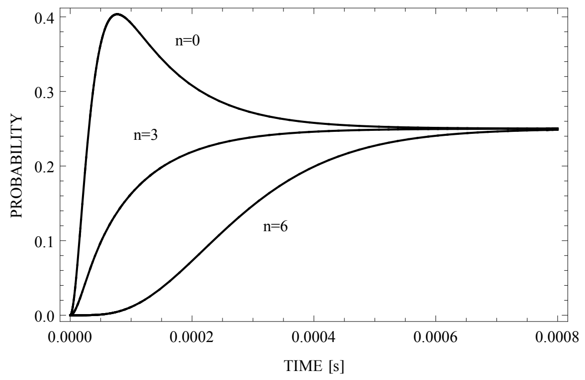

Fig. 1 Time-dependent probabilities $\mathbf{P}\{X(t)=1\,|\,X(0)=n\}$ for RED dropping function and different n's

For the GRED-type dropping function, defined as follows:

$$d_n = \begin{cases} 0, & n \le 1, \\ 0.2(n-1), & 1 < n \le 2, \\ 0.4n-0.6, & 2 < n < 4, \\ 1, & n \ge 4, \end{cases} \tag{24}$$

the results are presented in Fig. 2. Let us note that shapes of the corresponding distributions are similar (for RED and GRED types), however, of course, the stationary probabilities are different (they depend on the dropping function essentially).

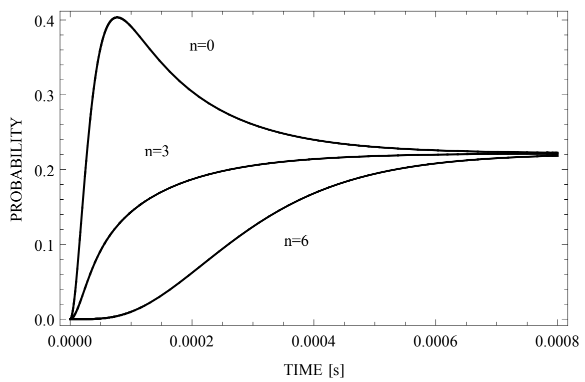

Fig. 2 Time-dependent probabilities $\mathbf{P}\{X(t)=1\,|\,X(0)=n\}$ for GRED dropping function and different n's

Analyze now the dependence of the time-dependent probability

$$\mathbf{P}\{X(0.01)=3\,|\,X(0)=1\}$$

on the analytical form of RED and GRED-type dropping functions for three different values of the traffic load ρ of the system. Assume, as previously, that the throughput of the output link equals 100 MB/s (we have $\mu = 15625$) and take the same buffer capacity (i.e. $K = 6$). Consider three different arrival rates of packets: 100, 120 and 140 Mb/s. Assuming that interarrival times have Erlang distributions with two degrees of freedom and with parameter λ we get, successively,

$$\lambda_1 = 31250, \quad \lambda_2 = 37500, \quad \lambda_3 = 43750.$$

Define the following family of RED-type dropping functions:

$$d_n^{(j)} = \begin{cases} 0, & n \le 1, \\ \frac{0.2(n-1)}{j}, & 1 < n < j+1, \\ 1, & n \ge j+1. \end{cases} \tag{25}$$

where $j = 1, ..., 5$.

In Table 1 we present the results of the experiment. Note that the greater traffic load ρ, the greater probability that the system is "half-saturated" ($X(0.01)=3$). Evidently, for successive RED functions (for $j = 1, ..., 5$) the proper probabilities decrease because the possibility of higher values of the queue size increases (the minimal queue size at which a packet can be physically dropped increases).

Table 1 Time-dependent probabilities $\mathbf{P}\{X(0.01)=3\,|\,X(0)=1\}$ for RED-type packet dropping

Dropping function	$\rho = 1$	$\rho = 1.2$	$\rho = 1.4$
RED-1	0.208198	0.285295	0.355553
RED-2	0.202148	0.248568	0.275993
RED-3	0.182720	0.200814	0.196934
RED-4	0.165368	0.162107	0.139443
RED-5	0.162630	0.156327	0.132085

One can make a similar observation for the following set of GRED-type dropping functions:

$$d_n^{(j)} = \begin{cases} 0, & n \le 1, \\ 0.2n-0.2, & 1 < n \le 2, \\ \frac{0.8(n-2)}{j}+0.2, & 2 < n < j+2, \\ 1, & n \ge j+2. \end{cases} \tag{26}$$

where $j = 1, ..., 4$.

In Table 2 we give transient probabilities $\mathbf{P}\{X(0.01)=3\,|\,X(0)=1\}$ for the same three levels of the traffic load ($\rho = 1, 1.2$ and 1.4).

Table 2 Time-dependent probabilities $\mathbf{P}\{X(0.01)=3\,|\,X(0)=1\}$ for GRED-type packet dropping

Dropping function	$\rho = 1$	$\rho = 1.2$	$\rho = 1.4$
GRED-1	0.208198	0.285295	0.355553
GRED-2	0.196057	0.256561	0.303277
GRED-3	0.188118	0.237794	0.269480
GRED-4	0.182573	0.224057	0.244528

Lastly, let us take into consideration the stationary queue-size distribution q_m ($m = 0, ..., 6$) in dependence on parameters of REM-type packet dropping. Introduce the following set of REM dropping functions:

$$d_n^{(j)} = \begin{cases} 0, & n \le j, \\ \frac{e^{n-j}-1}{e^{6-j}-1}, & j < n. \end{cases} \quad (27)$$

for $j = 1, ..., 5$.

Steady-state probabilities are given in Table 3 and visualized in Fig. 3 and Fig. 4, where cases of $j = 1$ and $j = 5$ (being, in fact, the classical Tail Drop algorithm) are presented, respectively.

Table 3 Stationary queue-size distribution for REM-type packet dropping

Queue size k	REM-1	REM-2	REM-3	REM-4	REM-5
0	0.040694	0.039586	0.037770	0.035188	0.032172
1	0.103805	0.101010	0.096399	0.089821	0.082128
2	0.131646	0.128248	0.122490	0.114189	0.104440
3	0.163702	0.162023	0.155184	0.144929	0.132693
4	0.193612	0.194577	0.194528	0.182859	0.168043
5	0.201008	0.204939	0.213301	0.225791	0.210341
6	0.142578	0.147293	0.159032	0.187384	0.252047

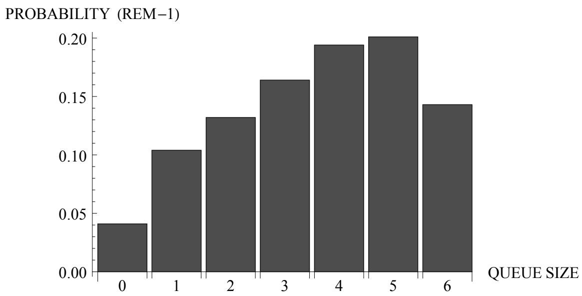

Fig. 3 Stationary queue-size distribution for REM-type dropping function ($j = 1$)

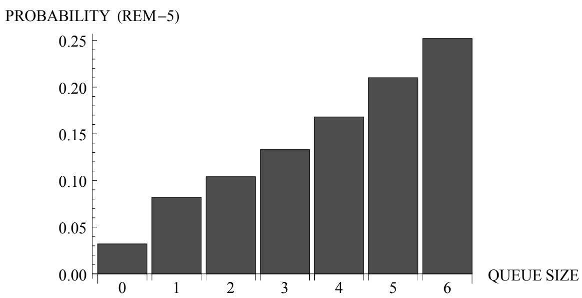

Fig. 4 Stationary queue-size distribution for REM-type dropping function ($j = 5$ – Tail Drop)

Let us observe that in the case of $j = 1$, in which the probability of dropping increases beginning with $n = 1$, the probability of buffer saturation is visibly lower as in the case of classical Tail Drop algorithm ($j = 5$), in which the packet is dropped only when the buffer is saturated.

5. CONCLUSIONS

In the paper a finite-buffer queueing model with general independent input stream of packets is investigated. The arrival process is controlled by an AQM-type dropping function deleting the incoming packets with probability depending on the queue size at the pre-arrival epoch. The idea of embedded Markov chain and the continuous law of total probability are applied to construct the system of Volterra-type integral equations for conditional queue-size distribution. The solution of the corresponding system written for Laplace transforms is obtained via linear algebraic approach. Network-motivated numerical examples with different dropping functions are attached as well.

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